37457

Advanced Bayesian Methods

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Major Goal for this Subject

By NEXT WEEK, tool you up to be able to do

ANY*

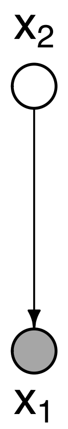
data analysis, no matter how complex,

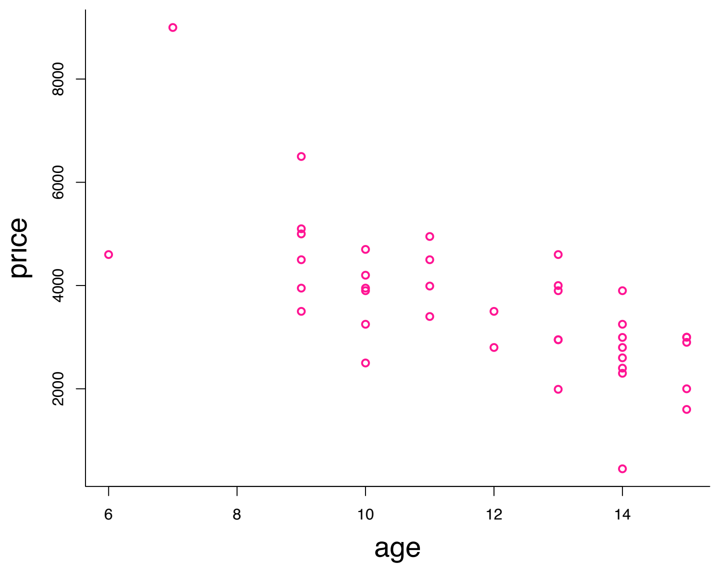
– and understand the underlying maths.

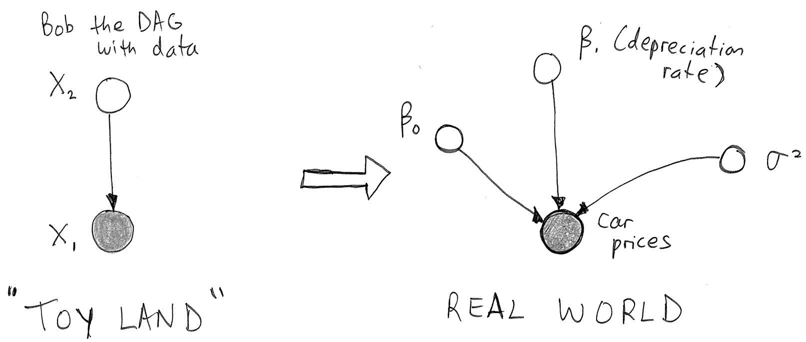
* To be qualified later.

From Toy Examples to Real Data Analysis

Bob the DAG with data







Bayesian Model For Mitsubishi Price/Age Data

$$\text{price}_i|\beta_0, \beta_1, \sigma^2 \;\sim\; N(\beta_0 + \beta_1\,\text{age}_i, \sigma^2)$$

$$p(\beta_0), \quad p(\beta_1), \quad p(\sigma^2),$$

often chosen to make 'prior' (pre-data) belief about these regression parameters non-informative.

The age values are treated as non-random here.

Interpretation of β_1:

$$\beta_1 = \text{annual depreciation rate of Mitubishi cars}$$

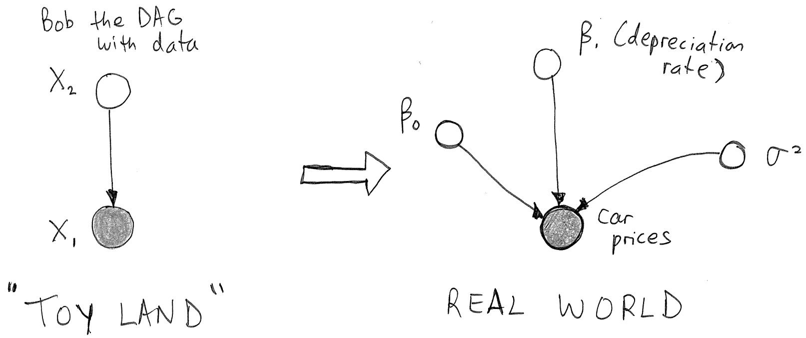
Posterior density function of β_1

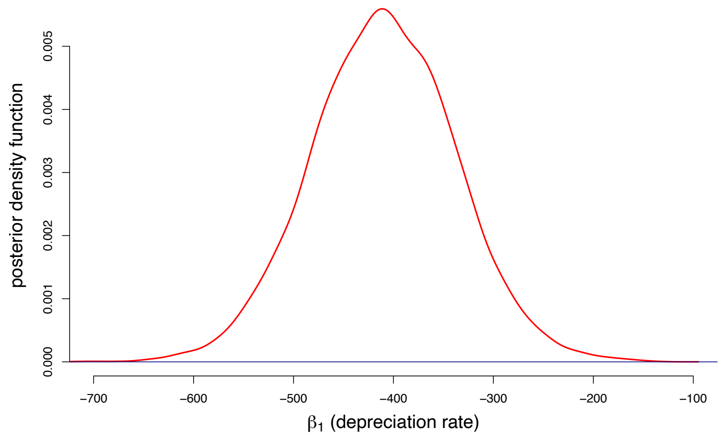
Needed:

$$p(\beta_1|\text{data on car prices})$$

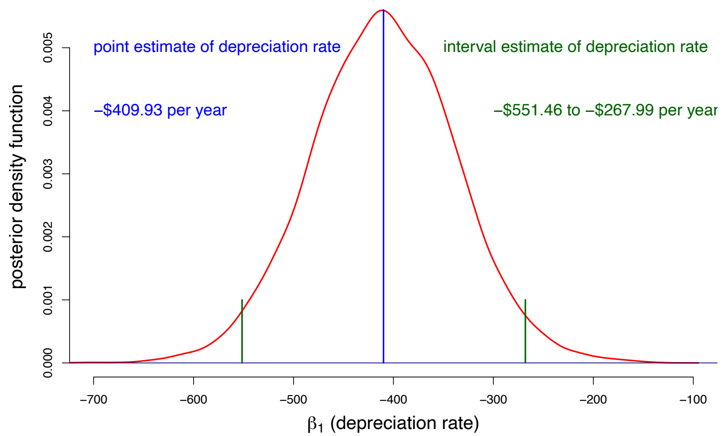
which is COMPLETELY ANALOGOUS to

$$p(x_2|\mathring{x}_1) \quad \text{in Assignment 2}$$





Show mitsubBayesJAGS.R script and run.



Inverse Gamma Notation

$$x \sim \text{Inverse-Gamma}(\kappa, \lambda)$$

$$\iff \quad p(x) = \frac{\lambda^\kappa}{\Gamma(\kappa)} x^{-\kappa-1} \exp(-\lambda/x), \quad x > 0.$$

Bayesian Inference for Normal Data

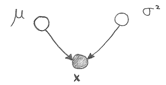
Consider the Bayesian version of inference for a univariate normal sample:

$$x_1, \ldots, x_n \overset{\text{ind.}}{\sim} N(\mu, \sigma^2).$$

A Bayesian model is:

$$x_i | \mu, \sigma^2 \overset{\text{ind.}}{\sim} N(\mu, \sigma^2)$$

$$\mu \sim N(0, \sigma_\mu^2), \quad \sigma^2 \sim \text{Inverse-Gamma}(\kappa_\sigma, \lambda_\sigma)$$



(Jessica the DAG again!)

We need:

$$p(\mu | \boldsymbol{x}) \quad \text{and} \quad p(\sigma^2 | \boldsymbol{x}).$$

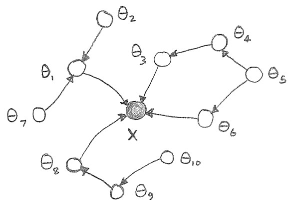
Bayesian Inference

Data: $\boldsymbol{x}$
Parameters: $\theta_1, \ldots, \theta_k$

Bayesian inference is based on the posterior density functions

$$p(\theta_1 | \boldsymbol{x}), \ldots, p(\theta_k | \boldsymbol{x})$$

But these are simply marginal conditional density functions as in Assignments 1 and 2.



Posterior Distributions

$$p(\mu|\boldsymbol{x}) = \frac{e^{-\mu^2/(2\sigma_\mu^2)}\Gamma(\kappa_\sigma + \frac{n}{2})}{\sqrt{2\pi}\sigma_\mu(\lambda_\sigma + \frac{1}{2}\|\boldsymbol{x} - \mu\mathbf{1}\|^2)^{\kappa_\sigma + \frac{n}{2}} \int_0^\infty (\sigma^2 + \sigma_\mu^2)^{-n/2}(\sigma^2)^{-\kappa_\sigma - 1} e^{-\frac{\|\boldsymbol{x}\|^2}{2(\sigma^2+\sigma_\mu^2)} - \frac{\lambda_\sigma}{\sigma^2}} d\sigma^2}$$

$$p(\sigma^2|\boldsymbol{x}) = \frac{(\sigma^2 + \sigma_\mu^2)^{-n/2}(\sigma^2)^{-\kappa_\sigma - 1} e^{-\frac{\|\boldsymbol{x}\|^2}{2(\sigma^2+\sigma_\mu^2)} - \frac{\lambda_\sigma}{\sigma^2}}}{\int_0^\infty (\sigma^2 + \sigma_\mu^2)^{-n/2}(\sigma^2)^{-\kappa_\sigma - 1} e^{-\frac{\|\boldsymbol{x}\|^2}{2(\sigma^2+\sigma_\mu^2)} - \frac{\lambda_\sigma}{\sigma^2}} d\sigma^2}$$

($\|\boldsymbol{v}\| = \sqrt{\boldsymbol{v}^T\boldsymbol{v}}$ is norm of the vector $\boldsymbol{v}$).

Even in the

univariate normal data setting

the posterior distributions involve

intractable integrals!!!

Central MCMC Theoretical Result

Theory says that successive draws from

$$p(\mu|\sigma^2, \boldsymbol{x}) \quad \text{and} \quad p(\sigma^2|\mu, \boldsymbol{x})$$

eventually leads to samples from

$$p(\mu, \sigma^2|\boldsymbol{x})!$$

This is known as Gibbs sampling

and is a special case of

Markov Chain Monte Carlo (MCMC)

How About the Full Conditionals?

The full conditionals in the univariate normal example are:

$$p(\mu|\sigma^2, \boldsymbol{x}) \quad \text{and} \quad p(\sigma^2|\mu, \boldsymbol{x})$$

$$p(\mu|\sigma^2, \boldsymbol{x}) \sim N\left(\frac{\sum_{i=1}^n x_i}{n + \sigma^2/\sigma_\mu^2}, \frac{\sigma^2}{n + \sigma^2/\sigma_\mu^2}\right)$$

$$p(\sigma^2|\mu, \boldsymbol{x}) \sim \text{Inverse-Gamma}\left(\kappa_\sigma + \tfrac{n}{2}, \lambda_\sigma + \tfrac{1}{2}\sum_{i=1}^n (x_i - \mu)^2\right)$$

Live R Demonstration

We will now do a live

MCMC

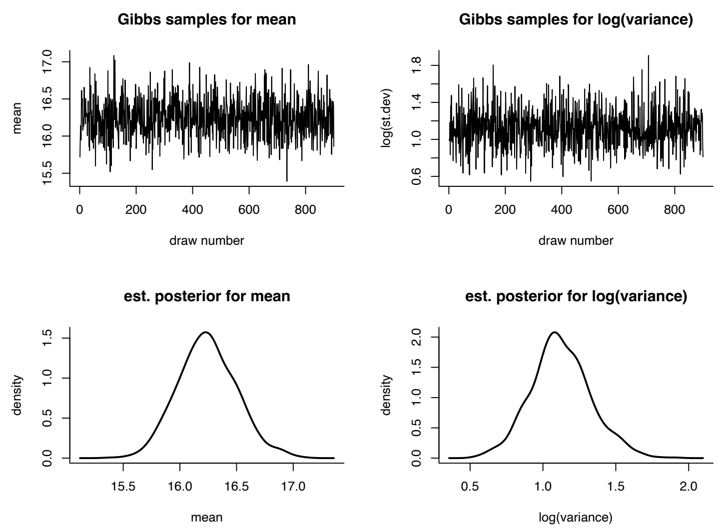
demonstration using

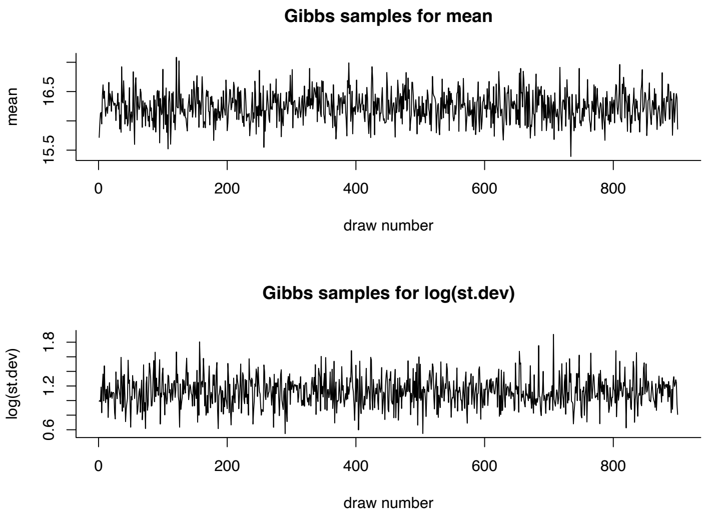
R

Illustration of MCMC

The following slides shows how MCMC works for the univariate normal example.

The first plots are successive draws from the full conditionals.





Practical MCMC

MCMC is a young (1990+) branch of Statistics and has several practical issues; e.g.

- partitioning of parameters,
- starting values,
- correlation between successive draws,
- convergence to required posteriors $\Longleftrightarrow$ length of burnin (also known as warmup).
- number of draws.

MCMC Software

Until late 2012, the most sophisticated MCMC software was from

The BUGS Project

BUGS is an acronym for

Bayesian inference Using Gibbs Sampling.

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JAGS is the same as BUGS but was introduced for Macintosh computer users since BUGS favoured Windows.

JAGS is an acronym for
Just Another Gibbs Sampler.

New Kid on the Block: Stan

2013 has seen the emergence of a new MCMC software product named

Stan

Starting in Laboratory 2 next week we will starting using and learning Stan via its R interface:

the rstan package

Please see the new link on the subject web-site under the heading

Advice about Laptop Preparation for Computer Laboratories
THIS IS NEEDED FOR NEXT WEDNESDAY'S LABORATORY 2

Class 6 Location

Recall that during Weeks 1 – 6 we have

ROOM CHANGES EVERY WEEK!

Class 6 is back in

Room 190, Level 4, Building 2
(where we were last week).

Assignment 4

is now on the
subject web-site.

Look at both JAGS and Stan script for mitsub example.

Some (Allowable) Screen Time!

SCREEN-FREE ZONE

REPRIEVE COMING UP

(you may start your screens!)

Task for You During the Break

Go to the web-site

http://distancecalculator.globefeed.com/australia_distance_calculator.asp

and determine

shortest road distance in kilometres from
your Sydney residence to UTS (to nearest 0.1km)

A good UTS address to use is:
15 Broadway, Ultimo, New South Wales
(which is near the street entrance to the UTS Tower)

First Up After the Break

Lecturer feedback on your Week 4 feedback.