

UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37457 Advanced Bayesian Methods

SOLUTIONS TO

ASSIGNMENT 7 (OPTIONAL)

$$1. \quad E(y|\beta, u) = \begin{bmatrix} E(y_{11}|\beta, u) \\ E(y_{12}|\beta, u) \\ E(y_{21}|\beta, u) \\ E(y_{22}|\beta, u) \\ E(y_{23}|\beta, u) \\ E(y_{31}|\beta, u) \\ E(y_{32}|\beta, u) \end{bmatrix}$$

$$= X\beta + Zu$$

$$= \begin{bmatrix} 1 & x_{11} \\ 1 & x_{12} \\ 1 & x_{21} \\ 1 & x_{22} \\ 1 & x_{23} \\ 1 & x_{31} \\ 1 & x_{32} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$= \begin{bmatrix} \beta_0 + \beta_1 x_{11} + u_1 \\ \beta_0 + \beta_1 x_{12} + u_1 \\ \beta_0 + \beta_1 x_{21} + u_2 \\ \beta_0 + \beta_1 x_{22} + u_2 \\ \beta_0 + \beta_1 x_{23} + u_2 \\ \beta_0 + \beta_1 x_{31} + u_3 \\ \beta_0 + \beta_1 x_{32} + u_3 \end{bmatrix}$$

Therefore, with these matrices,

$$E(y|\beta, u) = X\beta + Zu$$

is equivalent to

$$E(y_{ij}|\beta, u) = \beta_0 + \beta_1 x_{ij} + u_i, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n_i$$

and is verified here for $m=3$, $n_1=2$, $n_2=3$, $n_3=2$.

Hence, the first part of the Bayesian random intercepts model can be written succinctly using matrix notation as:

$$y|\beta, u, \sigma_\epsilon^2 \sim N(X\beta + Zu, \sigma_\epsilon^2 I).$$

2. The number of entries of Z_{grp} is 1003×423 .

Since all entries of Z_{grp} are either 0 or 1, the number of entries equal to 1 is the sum of the entries of Z_{grp} - which is 1003 (one for each row)

Therefore the number of entries equal to 0 is

$$1003 \times 423 - 1003 = 422 \times 1003.$$

The fraction of entries equal to 0 is

$$\frac{422 \times 1003}{423 \times 1003} = \frac{422}{423}.$$

As a percentage, this approximately

$$99.76\%.$$

Therefore Z_{gr} is very sparse, in

that only $\frac{1}{423} \approx 0.24\%$ of its

entries are non-zero.

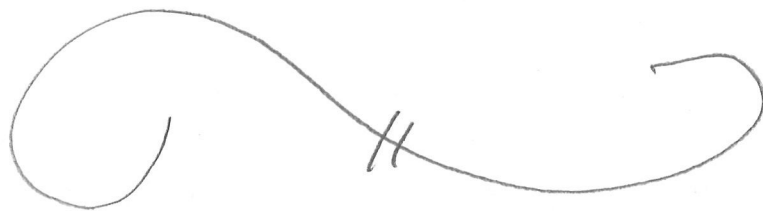
3. A correct version of the script

ozoneMidwestViaStan.R

is on the subject web-site under
the

Files for Assignment Solutions
heading.

The image plot of the
bivariate penalized splines fit to
the longitude, latitude and ozone
level data is on the next page.



latitude

