

UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37457 Advanced Bayesian Methods

SOLUTIONS TO

ASSIGNMENT 6 (OPTIONAL)

1(a) The x_i obtained with $x_i^{\text{orig}} = x_i^{\text{orig},F}$ is

$$\begin{aligned}
 x_i &= \frac{x_i^{\text{orig},F} - \bar{x}^{\text{orig},F}}{s_x^{\text{orig},F}} \\
 &= \frac{(1.8 x_i^{\text{orig},C} + 32) - \frac{1}{n} \sum_{i=1}^n (1.8 x_i^{\text{orig},C} + 32)}{\sqrt{\frac{1}{n-1} \sum_{i=1}^n \left\{ 1.8 x_i^{\text{orig},C} + 32 - \frac{1}{n} \sum_{i=1}^n (1.8 x_i^{\text{orig},C} + 32) \right\}^2}} \\
 &= \frac{1.8 (x_i^{\text{orig},C} - \bar{x}^{\text{orig},C}) + \cancel{32} - \frac{1}{n} (\cancel{n \times 32})}{\sqrt{\frac{1}{n-1} \sum_{i=1}^n \left\{ 1.8 (x_i^{\text{orig},C} - \bar{x}^{\text{orig},C}) + \cancel{32} - \frac{\cancel{n \times 32}}{n} \right\}^2}} \\
 &= \frac{\cancel{1.8} (x_i^{\text{orig},C} - \bar{x}^{\text{orig},C})}{\sqrt{\frac{\cancel{1.8}^2}{n-1} \sum_{i=1}^n (x_i^{\text{orig},C} - \bar{x}^{\text{orig},C})^2}} \\
 &= \frac{x_i^{\text{orig},C} - \bar{x}^{\text{orig},C}}{s_x^{\text{orig},C}}
 \end{aligned}$$

which is the x_i obtained with $x_i^{\text{orig}} = x_i^{\text{orig},C}$.

The x_i are the same, regardless of whether the original data are measured in Celsius or Fahrenheit.

I(b)

$$y_i \approx \beta_0 + \beta_1 x_i$$

$$\Rightarrow \frac{y_i^{\text{orig}} - \bar{y}^{\text{orig}}}{s_y^{\text{orig}}} \approx \beta_0 + \beta_1 \left(\frac{x_i^{\text{orig}} - \bar{x}^{\text{orig}}}{s_x^{\text{orig}}} \right)$$

$$\Rightarrow y_i^{\text{orig}} \approx \bar{y}^{\text{orig}} + s_y^{\text{orig}} \left(\beta_0 - \beta_1 \frac{\bar{x}^{\text{orig}}}{s_x^{\text{orig}}} \right)$$

$$+ s_y^{\text{orig}} \left(\frac{\beta_1}{s_x^{\text{orig}}} \right) x_i^{\text{orig}}$$

$$= \beta_0^{\text{orig}} + \beta_1^{\text{orig}} x_i^{\text{orig}}$$

where

$$\beta_0^{\text{orig}} \equiv \bar{y}^{\text{orig}} + s_y^{\text{orig}} \left\{ \beta_0 - \beta_1 (\bar{x}^{\text{orig}} / s_x^{\text{orig}}) \right\}$$

and

$$\beta_1^{\text{orig}} \equiv (s_y^{\text{orig}} / s_x^{\text{orig}}) \beta_1$$

Therefore, transformation (2) is required to ensure that the regression slope and intercept parameters correspond to the original units.

2. Please see the file
ratsModel3.R

under the heading

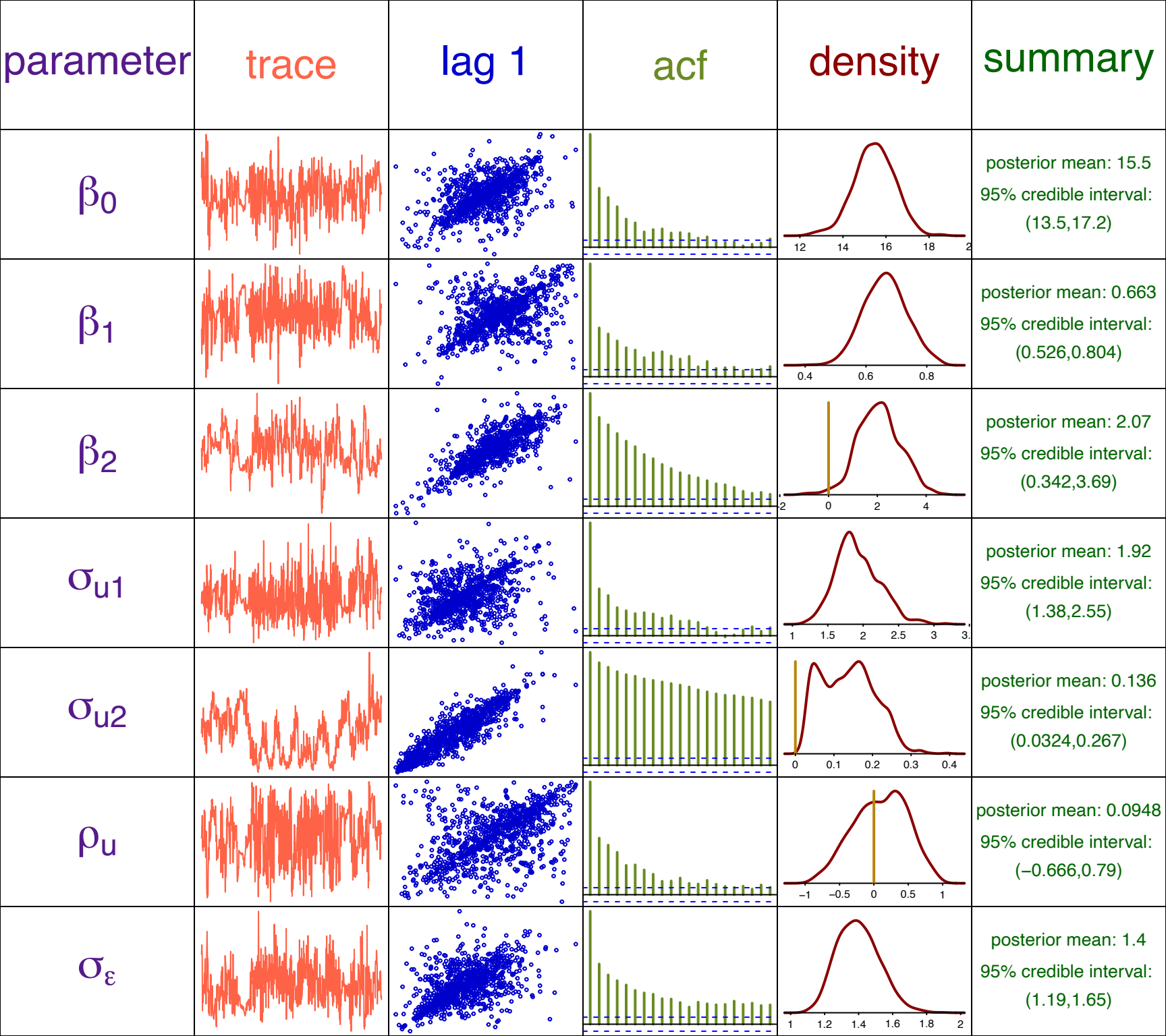
Files for Optional Assignments Solutions
on the subject web-site.

3. (a) Please see the file
orthodontModel2.R

under the heading

Files for Assignment Solutions

on the subject web-site. A Markov chain Monte Carlo summary plot produced by this file is on the next page. Note that answers differ due to sampling variability.



3(b) From the summary plot on the previous page

i. The Bayes estimate of β_2 is

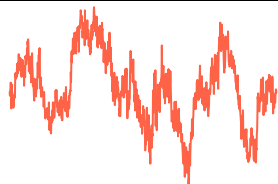
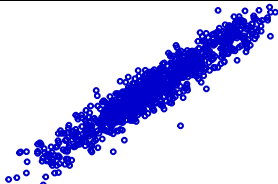
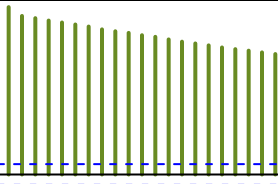
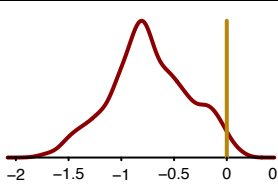
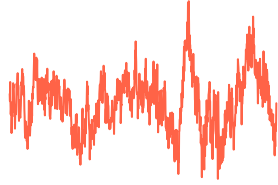
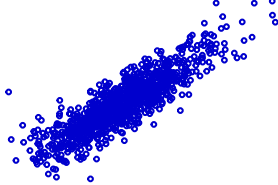
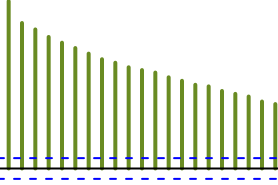
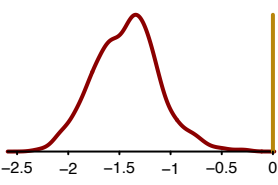
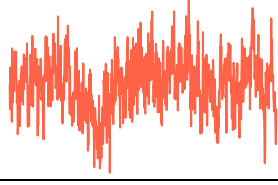
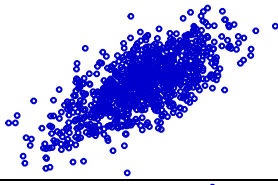
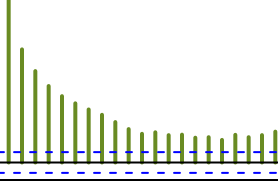
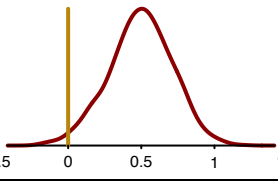
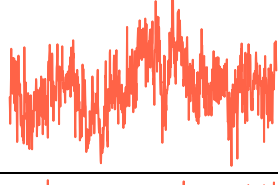
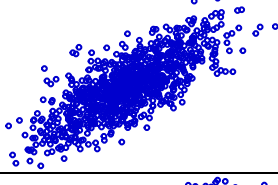
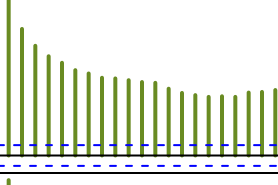
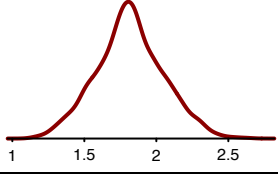
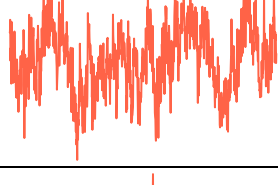
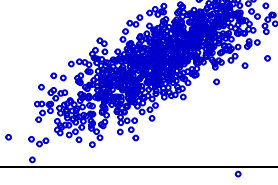
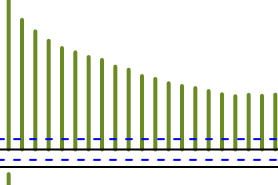
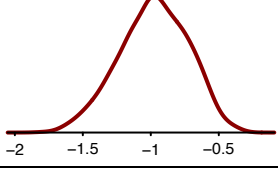
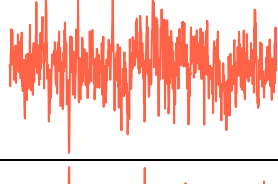
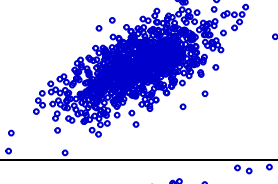
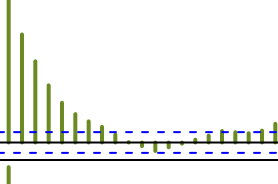
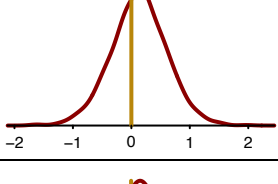
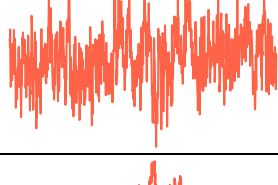
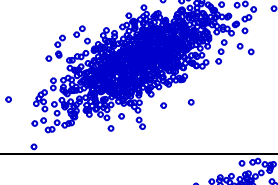
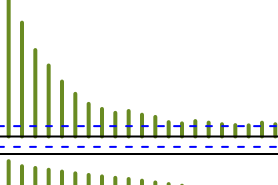
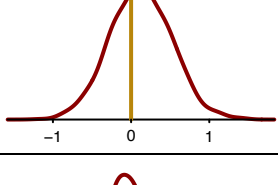
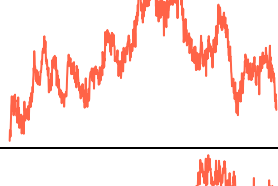
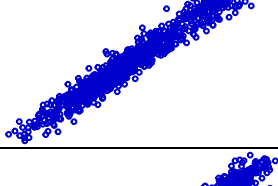
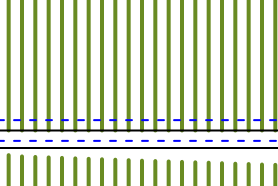
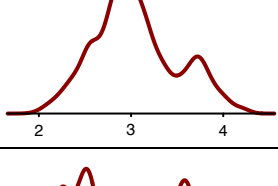
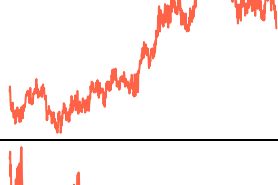
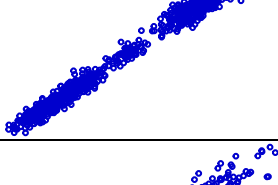
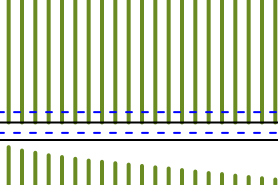
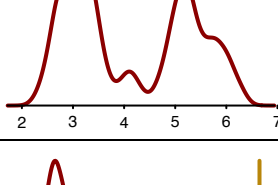
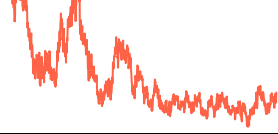
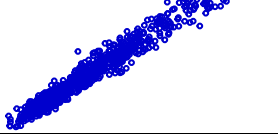
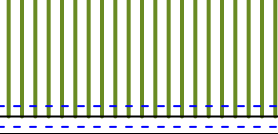
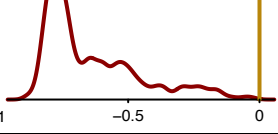
$$\hat{\beta}_{2, \text{Bayes}} = 2.07.$$

ii. A 95% credible interval for β_2 is

$$(0.342, 3.69).$$

NOTE WELL: These answers depend on the particular Markov chain Monte Carlo samples from the previous page. Different, but similar, answers arise for other versions of the samples - due to their randomness.

(c) Yes, since the answer to 3(b) ii does not contain zero.

parameter	trace	lag 1	acf	density	summary
intercept					posterior mean: -0.736 95% credible interval: $(-1.48, -0.0345)$
proportion indigenous in community					posterior mean: -1.43 95% credible interval: $(-2.05, -0.765)$
indicator of mother works					posterior mean: 0.496 95% credible interval: $(0.0443, 0.899)$
indicator of child aged over 2 years					posterior mean: 1.82 95% credible interval: $(1.36, 2.29)$
indicator of community in rural area					posterior mean: -0.978 95% credible interval: $(-1.49, -0.538)$
indicator of mother sec. school					posterior mean: 0.131 95% credible interval: $(-0.839, 1.06)$
indicator of husband sec. school					posterior mean: 0.124 95% credible interval: $(-0.63, 0.895)$
σ_{u0}					posterior mean: 3.05 95% credible interval: $(2.26, 3.96)$
σ_{u1}					posterior mean: 4.17 95% credible interval: $(2.51, 6.11)$
ρ_u					posterior mean: -0.641 95% credible interval: $(-0.835, -0.169)$

4. Based on the Markov chain Monte Carlo summary plot, 7 the proportion of indigenous in the community has a statistically significant negative impact on the probability of a child receiving a complete set of immunisations. The mother working and the child being over 2 years each have statistically significant positive impacts on this probability. Being in a rural area has a negative impact, which is statistically significant. The impacts of the other predictors are not statistically significant.

