

UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences
37457 Advanced Bayesian Methods
ASSIGNMENT 1

Alternative version with streamlined notation common in the Bayesian statistics literature.

Note: this not an assessment task and does not have to attempted (again).

1. Let x be a discrete random variable taking values 0 and 1 with equal probability and y be another random variable such that

$$\begin{aligned} p(y|0) &= \frac{y}{3}, \quad y = 1, 2, \\ p(y|1) &= \frac{5-y}{10}, \quad y = 1, 2, 3, 4. \end{aligned}$$

- (a) Construct a table for the joint probability mass function $p(x, y)$.
(b) Find $P(x + y \leq 2)$.
(c) Find $p(y)$, the marginal probability mass function of y .
2. Let discrete random variables x and y have joint probability mass function

$$p(x, y) = \begin{cases} \frac{40(3y+2)}{481(x^2+4)}, & x = 1, 2; \quad y = 5, 6 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal probability mass functions $p(x)$ and $p(y)$.

3. Let continuous random variables x and y have joint density function

$$p(x, y) = \begin{cases} \frac{y-x}{105}, & 2 < x < 5, \quad 5 < y < 12, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal density functions $p(x)$ and $p(y)$.

4. Let continuous random variables x and y have joint density function

$$p(x, y) = \begin{cases} \frac{e^{-y(x^2+1)}}{\pi}, & -\infty < x < \infty, \quad y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal density function $p(x)$.
(b) Determine the marginal density function $p(y)$.
(c) Determine $p(y|x = \overset{\circ}{x})$, the conditional density function of y given $x = \overset{\circ}{x}$.
5. Suppose that continuous random variables x and y have joint density function satisfying

$$p(x, y) \propto \exp\left(13xy - 94x^2 - \frac{1}{2}y^2\right), \quad -\infty < x < \infty, \quad -\infty < y < \infty.$$

(The \propto notation is relatively standard throughout the mathematical sciences and means that the left-hand side equals the right-hand side except for multiplicative factors that do not depend on the function arguments. For example, if $g(x, y) = 171 \cos(x + 12y)$ then we may write $g(x, y) \propto \cos(x + 12y)$.)

Determine $p(y|x = \overset{\circ}{x})$, the conditional density function of y given $x = \overset{\circ}{x}$.

