UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37457 Advanced Bayesian Methods

ABOUT YOUR FIRST ASSESSMENT TASK

Assignment 1, which commences overleaf, is the first assessment task for 37457 Advanced Bayesian Methods. It is worth 5% of the subject's total assessment. Please note that all assessment tasks are to be completed individually. There is no group work in 37457 Advanced Bayesian Methods. If you require some assistance with any of the assignments then please come to the help sessions that will be held on the afternoon before the due date. The details for the Assignment 1 help sessions are provided near the beginning of the assignment.

UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37457 Advanced Bayesian Methods

ASSIGNMENT 1

Due time and date: 10:05am, Wednesday 14th August, 2024.
Submission method: Hand to Professor Wand at start of Class 2.

NOTES:

- Handwritten solutions are acceptable and encouraged.
- All working should be shown.
- For the benefit of participants requiring assistance with this assignment, two optional-attendance help sessions will be as follows:

  Help Session 1: 3pm-4pm, Tuesday 13th August, 2024.

  Help Session 2: 4pm-5pm, Tuesday 13th August, 2024.

  Both of these help session will be held in Room 009A, Level 6, Building 7. **This room requires swipe card access through a sliding glass door near the Building 7 staircase. However, during normal work hours swipe card holders enter and exit every minute or two. If anyone questions you (unlikely) then please say that your entry is for a help session in our subject. The room is to the left and then ahead after entering the sliding door.**

1. Let X be a discrete random variable taking values 0 and 1 with equal probability and Y be another random variable such that

$$\begin{aligned} f_{Y|X}(y|0) &= \frac{y}{3}, \quad y = 1, 2, \\ f_{Y|X}(y|1) &= \frac{5-y}{10}, \quad y = 1, 2, 3, 4. \end{aligned}$$

   (a) Construct a table for the joint probability mass function $f_{X,Y}(x, y)$.

   (b) Find $P(X + Y \leq 2)$.

   (c) Find $f_Y(y)$, the marginal probability mass function of Y.

2. Let discrete random variables X and Y have joint probability mass function

$$f_{X,Y}(x, y) = \begin{cases} \dfrac{40(3y+2)}{481(x^2+4)}, & x = 1, 2; \quad y = 5, 6 \\ 0, & \text{otherwise.} \end{cases}$$

   Determine the marginal probability mass functions $f_X(x)$ and $f_Y(y)$.

3. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{y-x}{105}, & 2 < x < 5, \quad 5 < y < 12, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

4. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{e^{-y(x^2+1)}}{\pi}, & -\infty < x < \infty, \quad y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the marginal density function $f_X(x)$.

(b) Determine the marginal density function $f_Y(y)$.

(c) Determine $f_{Y|X}(y|x)$, the conditional density function of Y given $X = x$.

5. Suppose that continuous random variables X and Y have joint density function satisfying

$$f_{X,Y}(x,y) \propto \exp\left(13xy - 94x^2 - \tfrac{1}{2}y^2\right), \quad -\infty < x < \infty, \ -\infty < y < \infty.$$

(The $\propto$ notation is relatively standard throughout the mathematical sciences and means that the left-hand side equals the right-hand side except for multiplicative factors that do not depend on the function arguments. For example, if $g(x,y) = 171\cos(x + 12y)$ then we may write $g(x,y) \propto \cos(x + 12y)$.)

Determine $f_{Y|X}(y|x)$, the conditional density function of Y given $X = x$.

♦ ♦ ♦