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Advanced Bayesian Methods

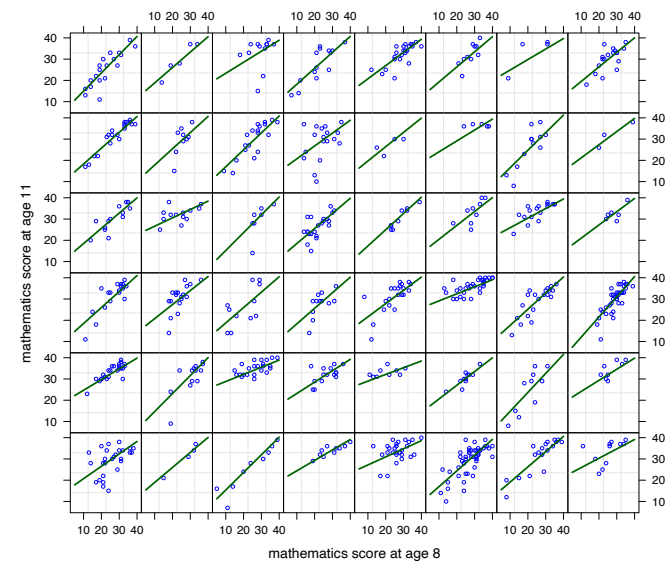
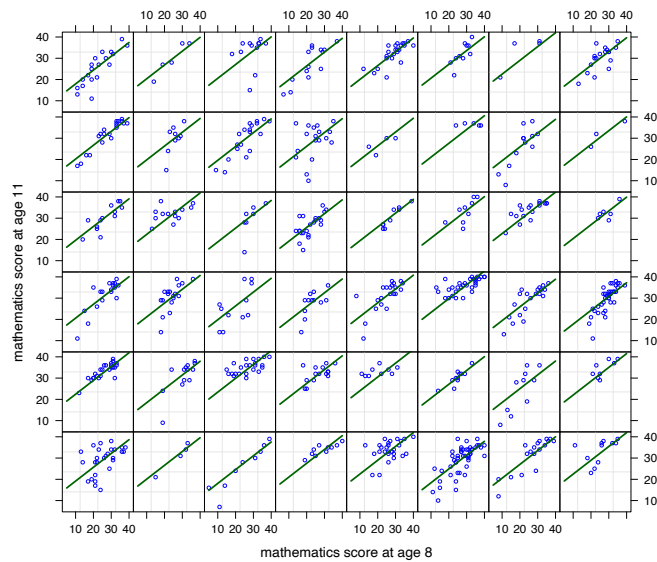
Bayesian Models for Grouped Data: Additional Aspects

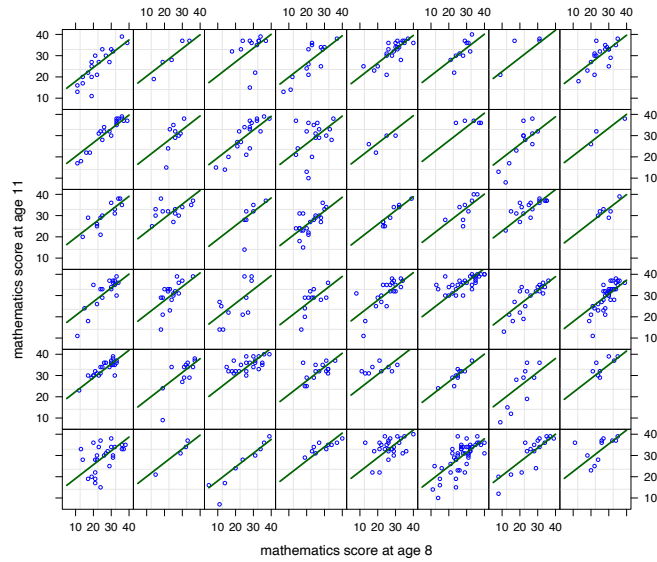
The previous slide fit the Bayesian random intercepts model

$$y_{ij} | \beta_0, \beta_1, u_i, \sigma_\varepsilon^2 \text{ ind.} \sim N\left((\beta_0 + u_i) + \beta_1 x_{ij}, \sigma_\varepsilon^2\right)$$

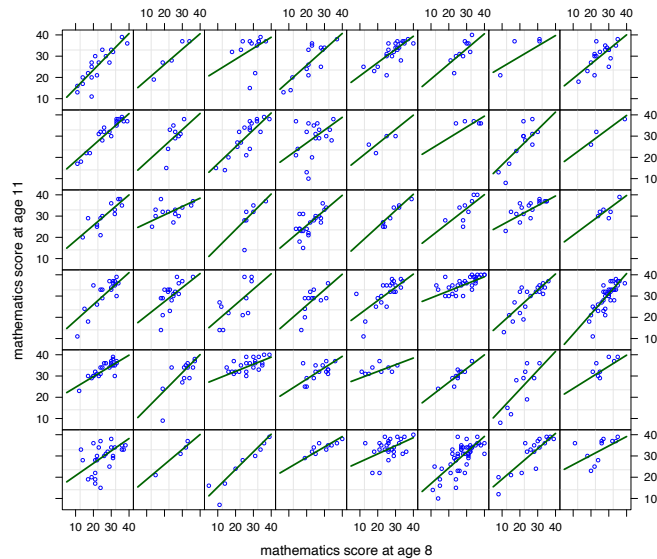
$$u_i | \sigma_u^2 \text{ ind.} \sim N(0, \sigma_u^2)$$

LIMITATION: The lines for each school are **parallel** (i.e. same slope).





Random Intercepts and Slopes Models



$$y_{ij} | \beta_0, \beta_1, u_{0i}, u_{1i}, \sigma_\varepsilon^2 \stackrel{\text{ind.}}{\sim} N\left((\beta_0 + u_{0i}) + (\beta_1 + u_{1i})x_{ij}, \sigma_\varepsilon^2\right),$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} \left| \sigma_{u0}^2, \sigma_{u1}^2, \rho_u \stackrel{\text{ind.}}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{u0}^2 & \rho_u \sigma_{u0} \sigma_{u1} \\ \rho_u \sigma_{u0} \sigma_{u1} & \sigma_{u1}^2 \end{bmatrix}\right).$$

Note that

$$\begin{bmatrix} \sigma_{u0}^2 & \rho_u \sigma_{u0} \sigma_{u1} \\ \rho_u \sigma_{u0} \sigma_{u1} & \sigma_{u1}^2 \end{bmatrix}$$

is a general 2×2 covariance matrix where

$-1 < \rho_u < 1$ is the correlation between the u_{0i} and u_{1i} .

Variance Component Prior Debate (early 2000s)

Look at Stan code in `randIntAndSlpViaStan.R`

Until about 2000 most Bayesian analysts used

$$p(\sigma^2) \sim \text{Inverse-Gamma}(\varepsilon, \varepsilon) \quad (\varepsilon \text{ 'small'})$$

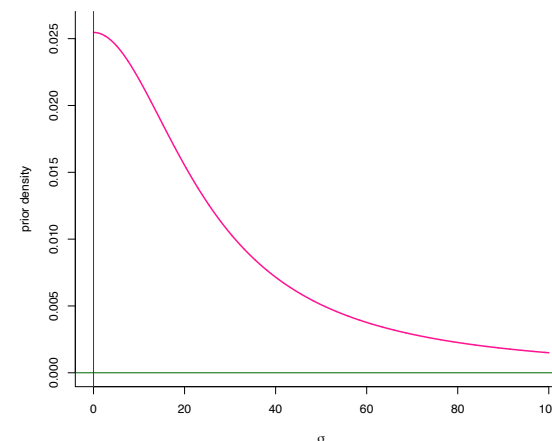
as a 'vague' prior for variance components – because of its conjugate status.

Recent literature (e.g. [Gelman, 2006, *Bayesian Analysis*](#)) has criticised this choice since the $\text{Inverse-Gamma}(\varepsilon, \varepsilon)$ distribution is not very vague.

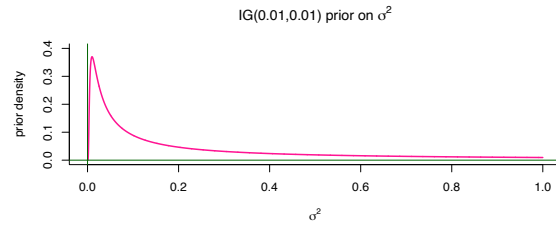
Gelman's default recommendation is **half-Cauchy with high scale parameter**.

$$p(\sigma) = \frac{2A}{\pi(\sigma^2 + A^2)}, \quad \sigma^2 > 0, \quad A \text{ 'big'}$$

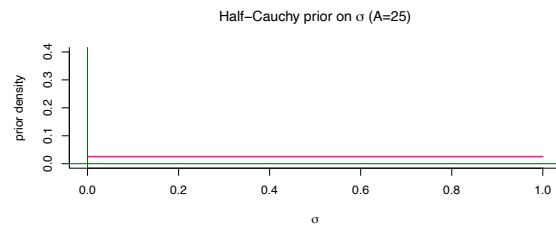
Half-Cauchy prior on σ ($A=25$)



HALF-CAUCHY PRIORS FOR STANDARD DEVIATION PARAMETERS



Look at Stan code



Covariance Matrix Priors

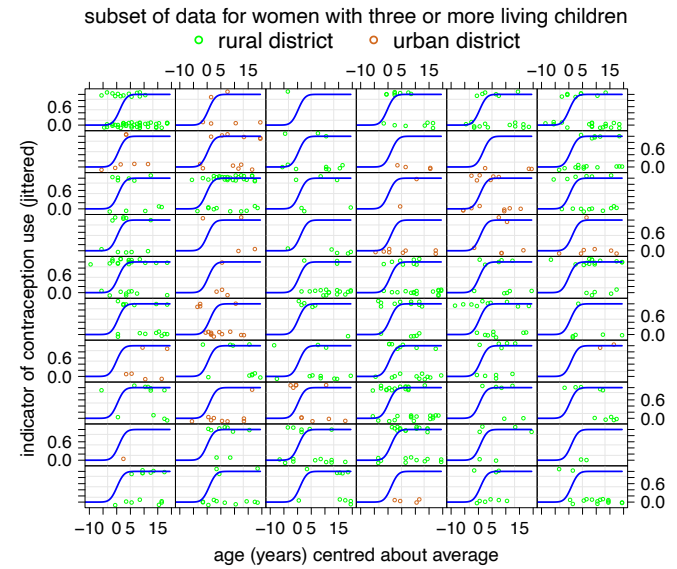
In 2012 (here in Ultimo, N.S.W.) Alan Huang and Matt Wand extended the Gelman Half-Cauchy prior idea to covariance matrix priors and published:

Huang, A. and Wand, M.P. (2013). Simple marginally noninformative prior distributions for covariance matrices. *Bayesian Analysis*, **8**, 439–452.

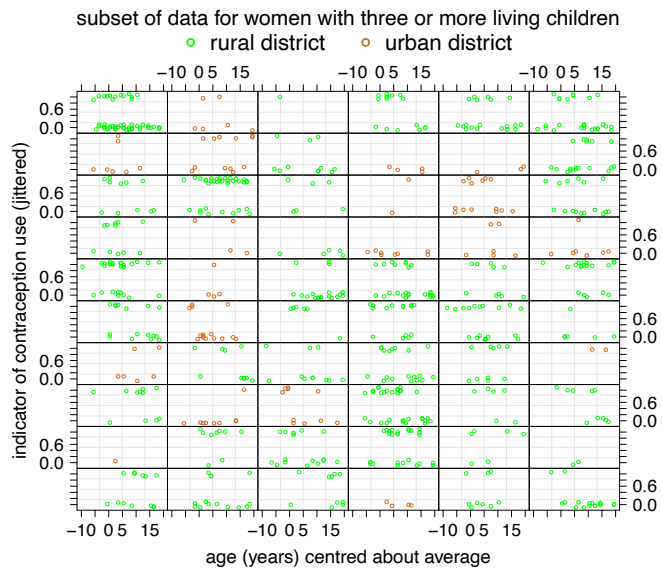
(Naturally) the class's Stan scripts use this prior – but technical details left aside (but Huang & Wand paper on lecturer's web-pages).

BINARY RESPONSE

GROUPED DATA



Logistic Mixed Models



An example is:

$$y_{ij} | \beta_0, \beta_1, \beta_2, \beta_3 \stackrel{\text{ind.}}{\sim} \text{Bernoulli} \left(\frac{1}{1 + \exp[-\{\beta_0 + u_{i0} + (\beta_1 + u_{i1})x_{1ij} + \beta_2 x_{2i}\}]} \right),$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} \Bigg| \sigma_{u0}^2, \sigma_{u1}^2, \rho_u \stackrel{\text{ind.}}{\sim} N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{u0}^2 & \rho_u \sigma_{u0} \sigma_{u1} \\ \rho_u \sigma_{u0} \sigma_{u1} & \sigma_{u1}^2 \end{bmatrix} \right)$$

Assignment 6, Question 4 has an example of this type.

Let's look at coding in Stan...

Markov Chain Monte Carlo Making a Difference

Without Markov chain Monte Carlo, fitting logistic mixed models with accurate inference is **VERY HARD** due to intractable integrals (even in the non-Bayesian case).

The availability of Bayesian inference engines makes such analyses **routine** – but this is **quite a recent development** for such an important model.

