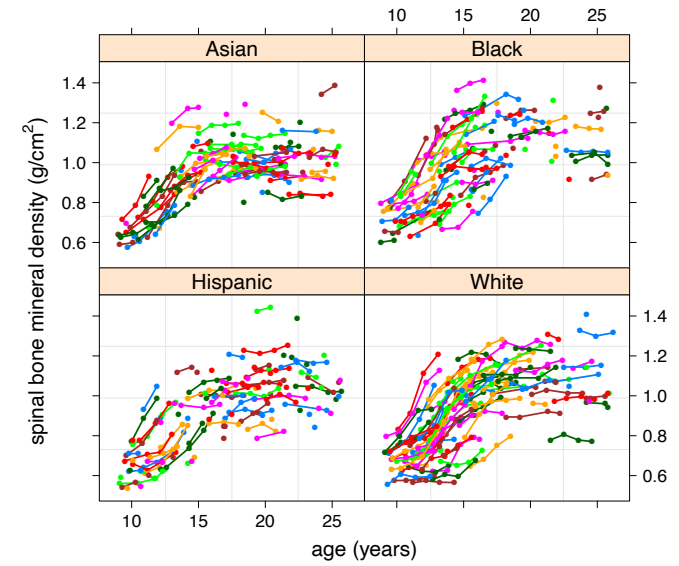


37457

Advanced Bayesian Methods

# Some Generalized Additive Models Extensions

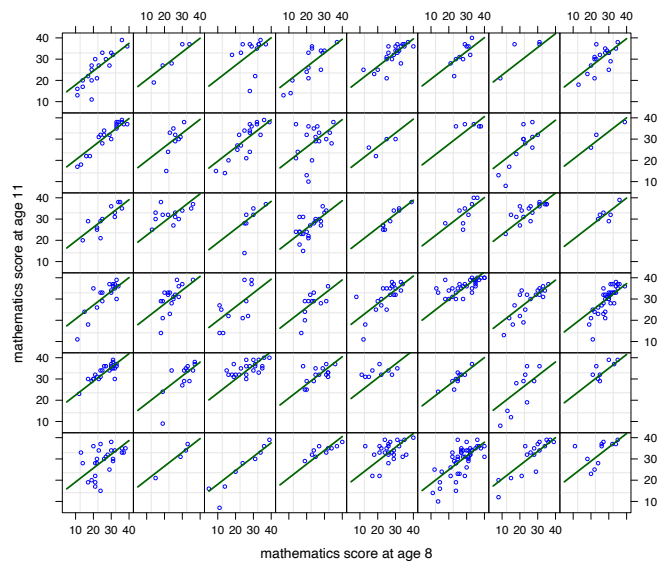


## Generalized Additive Mixed Model Extension

SBMD = spinal bone mineral density

Ordinary generalized additive model:

$$SBMD_i = f(\text{age}_i) + \beta_2 \text{Black}_i + \beta_3 \text{Hispanic}_i + \beta_4 \text{White}_i + \varepsilon_i$$



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But this ignores the longitudinal (repeated measures) aspect of the data.

↓

Generalized additive mixed model:

$$\text{SBMD}_{ij} = u_{\text{grp},i} + f(\text{age}_{ij}) + \beta_2 \text{Black}_i + \beta_3 \text{Hispanic}_i + \beta_4 \text{White}_i + \varepsilon_{ij},$$

$$u_{\text{grp},i} \stackrel{\text{ind.}}{\sim} N(0, \sigma_{\text{grp}}^2).$$

## Treating Grouped and Penalized Splines Together

$$Z = \left[ \begin{array}{c|c} \text{random} & \text{spline} \\ \text{intercept} & \text{basis} \\ \text{indicators} & \text{functions} \end{array} \right]$$

$$\text{covariance matrix of random effects vector} = \begin{bmatrix} \sigma_{\text{grp}}^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_{\text{spl}}^2 \mathbf{I} \end{bmatrix}$$

$\sigma_{\text{grp}}^2$  = controls between-group variability

$\sigma_{\text{spl}}^2$  = controls amount of spline penalization

## Bivariate Function Extensions

Generalized additive model with three predictors:

$$y_i = \beta_0 + f_1(x_{1i}) + f_2(x_{2i}) + f_3(x_{3i}) + \varepsilon_i$$

A bivariate function extension:

$$y_i = \beta_0 + f_{12}(x_{1i}, x_{2i}) + f_3(x_{3i}) + \varepsilon_i$$

Often (but not necessarily) the  $(x_{1i}, x_{2i})$  data correspond to geographical position (e.g. (longitude,latitude)).

This extension also known as a **geoadditive model**.

## Sydney Real Estate Example

The HRW package in R has a demonstration named

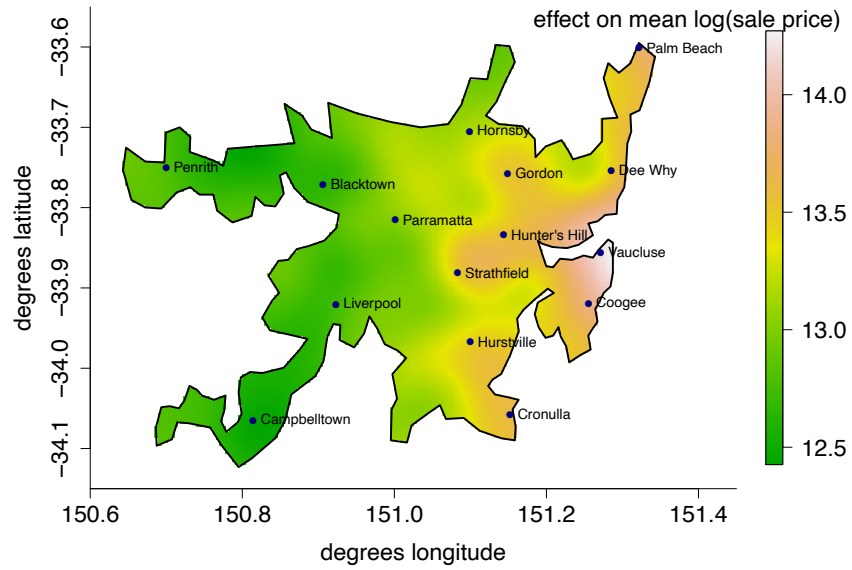
SydneyDisplaySophs

that fits such a geoadditive model to data from

37,676 houses in **OUR CITY**.

See **Assignment 7** for further details and Stan implementation.

## Penalized Splines for Bivariate Functions



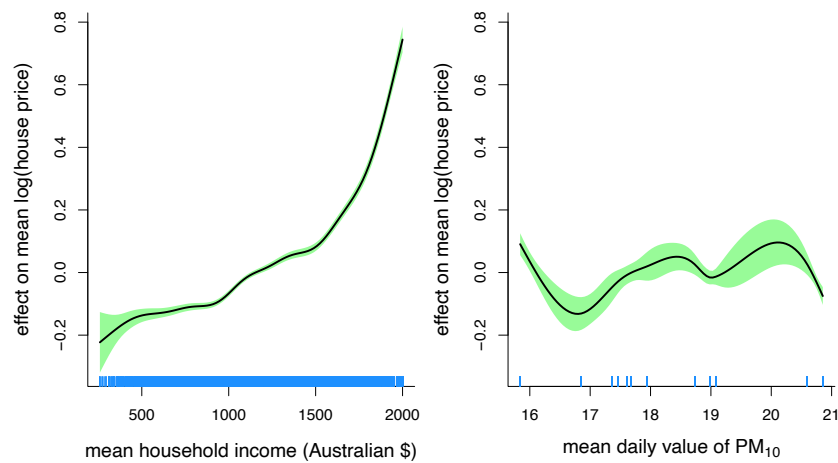
Note to presenter: Run scallopFull.Rs (in sprSC)

## Bivariate Penalized Splines in Assignment 7

See [Assignment 7](#) for further details and Stan implementation of:

bivariate penalized splines

(including [very cool 3D spin graphics](#)).



## WHAT'S LEFT TO DO IN 37457?

Assignment 7 (now on web-site; due next week)

Second last topic: Bayesian model selection → Assignment 8 (due Class 12)  
(this starts after the break)

Last topic: "Non-clean" data (Class 11) → Laboratory 4 (Class 12)

Practice final exam (handed out in Class 12) Practice final exam solutions  
(will be put on web-site)

Final exam on 13th November Receive your grade (December)

