UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37457 Advanced Bayesian Methods

CLASS 8 OPTIONAL EXTRA QUANTITATIVE FINANCE EXAMPLES

1 Introduction

These are some brief notes to accompany the Class 8 quantitative finance examples. Note that this material is not formally part of 37457 Advanced Bayesian Methods, but has been inserted into the pre-break hour of the subject. This insertion recognises that roughly half of the 2024 class participants are enrolled in quantitive finance and related topics.

The examples have been borrowed from some web-sites maintained by Professor Hedibert Freitas Lopes, Insper Institute of Education and Research, Brazil.

2 Illustrative Data Set

This document's illustrative data is based on the following raw time series data set:

$$p_t = \text{closing price of the Deutscher Aktien index (German stock index, with acronym “DAX”) on day } t$$

for 1860 consecutive trading days during 1991–1999. Let

$$r_t \equiv \log\left(\frac{p_t}{p_{t-1}}\right) = \text{the } \textit{return} \text{ on day } t.$$

Then the *mean-centred returns* are

$$y_t \equiv r_t - \overline{r} \quad \text{where} \quad \overline{r} \equiv \text{average of the } r_t\text{'s.}$$

Let T denote the number of y_t values.

The next two sections describe Bayesian models for these mean-centred returns.

3 A Bayesian Generalized Autoregressive Heteroscedascity Model

A Bayesian *generalized autoregressive heteroscedascity order* $(1,1)$ model (usually shortened to GARCH$(1,1)$ model) for the mean-centred returns is

$$y_t|\mu,\sigma_t \sim N(\mu,\sigma_t^2), \quad 1 \le t \le T$$

where

$$\sigma_t^2 = \alpha_0 + \alpha_1(y_{t-1}-\mu)^2 + \beta_1\sigma_{t-1}^2, \quad 2 \le t \le T,$$

with

$$\sigma_1 \equiv \text{sample standard deviation of the first 10 } y_t \text{ values.}$$

There are many possible prior distributions for the model parameters. In the Class 8 illustration we use

$$\mu \sim N(0, 100), \quad \alpha_0 \sim N(0, 100), \quad \alpha_1 \sim N(0, 100)$$

and

$$\beta_1 | \alpha_1 \sim N(0, 100) \quad \text{truncated to the interval } [1 - \alpha_1, \infty).$$

Models of this type were first formulated by the American econometrician Robert F. Engle. An early paper is Engle (1982). In 2003 Professor Engle was awarded the Nobel Prize in Economics for these contributions.

4 A Bayesian Stochastic Volatility Model

Let $t_\nu(\mu, \sigma^2)$ denote the t-distribution with ν degrees of freedom, location parameter μ and scale parameter σ. A Bayesian *stochastic volatility* model for the mean-centred returns is

$$y_t | h_t \overset{\text{ind.}}{\sim} t_\nu(0, \exp(h_t)), \quad 1 \le t \le T,$$

where

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma\, u_t, \quad 2 \le t \le T,$$

$$h_1 | \mu, \sigma, \phi \sim N\left(\mu, \frac{\sigma^2}{1 - \phi}\right), \quad u_t \overset{\text{ind.}}{\sim} N(0, 1), \quad 1 \le t \le T.$$

There are many possible prior distributions for the model parameters. In the Class 8 illustration we use

$$\mu \sim N(0, 100), \quad \sigma^2 \sim \text{Gamma}(\tfrac{1}{2}, \tfrac{1}{2}), \quad \nu \sim \text{Gamma}(1, 0.05)$$

and

$$\phi \sim N(0.8, 0.09) \quad \text{truncated to the interval } [-1, 1].$$

Further details concerning this model can be found in e.g. Shephard (2010).

5 Other Quantitative Finance Models

2024 is the first year that 37457 Advanced Bayesian Methods has had more than a few quantitive finance participants. If any of the class participants have suggestions for other quantitative finance models and/or data sets to look into then please pass them onto the lecturer. There may be some pockets of time in the remaining four classes to look into their Bayesian inference engine-based analyses.

References

Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, **50**, 987–1007.

Shephard, N. (2010). Stochastic volatility models. In *Macroeconometrics and Time Series Analyses* (pp. 276–287). London: Palgrave Macmillan, United Kingdom.