BAYESIAN INFERENCE FOR
p = probability of a human being left-handed
BASED ON OUR CLASS 4 DATA SET

$$n = 11$$

$$\sum_{i=1}^{n} X_i = 0$$

$$\text{Beta}\left(2 + \sum_{i=1}^{n} X_i,\ 18 + n - \sum_{i=1}^{n} X_i\right) = \text{Beta}(2, 29)$$

$$\hat{p}_{\text{Bayes}} = \frac{2}{2+29} = \frac{2}{31} \simeq 0.06452$$

$$\hat{p}_{\text{freq.}} = \frac{0}{11} = 0$$

$$0.025 = P(0 < p < L \mid X_1, \dots, X_n)$$

$$= \int_0^L \frac{\Gamma(2)\,\Gamma(29)}{\Gamma(31)}\, p^{2-1} (1-p)^{29-1}\, dp$$

$$= F(L; 2, 29)$$

where $F(\cdot\,; 2, 29)$ is the cumulative distribution function of the Beta$(2,29)$ distribution.

quantile function

$$\Rightarrow L = F^{-1}(0.025; 2, 29) = Q(0.025; 2, 29)$$

$$= \text{qbeta}(0.025, 2, 29) \text{ in R}$$

$$\simeq 0.008178$$

Similarly $U = \text{qbeta}(0.975; 2, 29)$

$$\simeq 0.1722$$

$\Rightarrow$ a 95% credible interval for p is $(0.008178, 0.1722)$.