The Resultant Markov Chain Monte Carlo Algorithm

The full set of full conditional distributions is:

$$\begin{aligned}
x_1|x_2, x_3 &\sim N\left(\frac{12 - 22x_3(x_2 - 47)}{2(121x_3 + 2)}, \frac{1}{2(121\, x_3 + 2)}\right) \\
x_2|x_1, x_3 &\sim N(47 - 11x_1, 1/(2x_3)), \\
x_3|x_1, x_2 &\sim \text{Gamma}\left(\tfrac{13}{2}, (x_2 - 47 + 11\, x_1)^2 + 1\right).
\end{aligned}$$

The resultant Markov chain Monte Carlo algorithm with *burnin* size B and *kept* size K is as follows:

Initialize: $x_2^{[0]}$, $x_3^{[0]}$

Cycle: $g = 1, \ldots, B + K$:

$$x_1^{[g]} \sim N\left(\frac{12 - 22x_3^{[g-1]}(x_2^{[g-1]} - 47)}{2(121x_3^{[g-1]} + 2)}, \frac{1}{2(121\, x_3^{[g-1]} + 2)}\right),$$

$$x_2^{[g]} \sim N\left(47 - 11x_1^{[g]}, 1/(2x_3^{[g-1]})\right),$$

$$x_3^{[g]} \sim \text{Gamma}\left(\tfrac{13}{2}, (x_2^{[g]} - 47 + 11\, x_1^{[g]})^2 + 1\right).$$

Discard the burnin samples: $x_1^{[g]}$, $x_2^{[g]}$, $x_3^{[g]}$ for $1 \le g \le B$.

Assuming that B is large enough to ensure convergence, the kept samples:

$$x_1^{[g]},\ x_2^{[g]},\ x_3^{[g]} \text{ for } B + 1 \le g \le B + K$$

are samples from the marginal distributions corresponding to $p(x_1)$, $p(x_2)$ and $p(x_3)$.