UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET L

THEME: **Integration Methods**

Date of this version: **10th July 2026**[1]

Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

L.1. Evaluate the integral

$$\int_0^\infty \exp(-7x^4)\,dx.$$

L.2. Evaluate the integral

$$\int_0^1 x^{-1/8}(1-x)^{-7/8}\,dx.$$

L.3. Evaluate the Cauchy principle value integral

$$\mathrm{pv}\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx.$$

L.4. Show that

$$\int_1^\infty \frac{\ln x}{x^2+1}\,dx = G$$

where G is Catalan's constant. The following result concerning interchange of integration and limits may be assumed:

$$\lim_{n\to\infty}\int_1^\infty f_n(x)\,dx = \int_1^\infty \lim_{n\to\infty} f_n(x)\,dx$$

where

$$f_n(x) = \sum_{k=0}^{n} (-1)^k \ln(x) x^{-2k-2} \quad \text{for all } n\in\mathbb{N} \text{ and } x>1.$$

[1]This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

L.5. Evaluate

$$\int_0^1 \frac{x\ln^3(x)}{1-x^4}\,dx.$$

The fact that all relevant integrand functions are continuous and Riemann integrable may be assumed. In addition, it may be assumed that extensions of Theorems 5.9 and 5.10 to the interval $[0,\infty)$ hold for the relevant sequences of functions.

L.6. Evaluate

$$\int_0^1 \frac{\ln^5(x)}{x+1}\,dx.$$

The following result concerning interchange of integration and limits may be assumed:

$$\lim_{n\to\infty}\int_0^1 f_n(x)\,dx = \int_0^1 \lim_{n\to\infty} f_n(x)\,dx$$

where

$$f_n(x) = \sum_{k=0}^{n} \ln^5(x)(-x)^k \quad \text{for all } n\in\mathbb{N} \text{ and } 0<x<1.$$

L.7. By consideration of the function

$$f(x,t) = \frac{x^t-1}{\ln(x)}, \qquad x,t>0,$$

and Leibniz's rule, evaluate

$$\int_0^1 \frac{x^{\sqrt{719\,\pi^3}}-1}{\ln(x)}\,dx.$$

L.8. By consideration of the function

$$f(x,t) = \frac{\ln\big(1+tx\big)}{1+x^2}, \qquad x\in(0,1),\ t>0,$$

and Leibniz's rule, evaluate

$$\int_0^1 \frac{\ln(1+x)}{1+x^2}\,dx.$$

L.9. For all $t\in\mathbb{R}$ let

$$F(t) = \int_{t^2}^{2\cosh(t)} e^{-x(t^4+8)}\,dx.$$

Use Leibniz's rule to evaluate $F'(t)$.

L.10. By consideration of the function

$$f(x,t) = \frac{\cos(tx)}{1+x^2}, \qquad x,t\in\mathbb{R},$$

and Leibniz's rule, evaluate

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{1+x^2}\, dx.$$

The facts that

$$\int_0^{\infty} \frac{\sin(x)}{x} dx = \frac{\pi}{2}$$

and the differential equation

$$y = \frac{d^2y}{dt^2} \quad \text{has the solutions} \quad y = C_1 e^t + C_2 e^{-t} \quad \text{for all} \quad C_1, C_2 \in \mathbb{R}$$

may be assumed.

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