

UNIVERSITY OF TECHNOLOGY SYDNEY  
School of Mathematical and Physical Sciences  
**35007 Real Analysis**

EXERCISE SET H

**THEME: L'Hôpital's Rule**

Date of this version: 10th July 2026<sup>1</sup>

*Solutions? Full solutions to all exercises will be released after the relevant class and/or tutorial.*

H.1. Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan^{-1}(109x)}{42x + 71x\sqrt{x}}.$$

H.2. Evaluate

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{\ln(x^3 + x)}.$$

H.3. Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin^2(x)}{\tan(x^2)}.$$

H.4. Evaluate

$$\lim_{x \rightarrow 16348} \frac{x^{1/14} - 2}{x - 16348}.$$

H.5. Evaluate

$$\lim_{x \rightarrow 0+} x \ln(x).$$

H.6. Evaluate

$$\lim_{x \rightarrow 0} \frac{e^x \sin(x) - x(1 + x)}{x^3}.$$

H.7. Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin^{947}(x)}{\sin(x^{947})}.$$

H.8. Evaluate

$$\lim_{x \rightarrow 11+} \frac{\ln(x - 11)}{\ln(x^2 - 121)}.$$

---

<sup>1</sup>This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

H.9. Evaluate

$$\lim_{x \rightarrow 0} \frac{39^x - 22^x}{x}.$$

H.10. Evaluate

$$\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))}{\sqrt{x}}.$$

H.11. Evaluate

$$\lim_{x \rightarrow 1+} \{(x-1) \tan(\pi x/2)\}.$$

H.12. Evaluate

$$\lim_{x \rightarrow \infty} \left( \frac{x^4}{x^2 - 1} - \frac{x^4}{x^2 + 1} \right).$$

H.13. Which is the correct interpretation of  $0^0$ ,

$$0^0 = 0 \quad \text{or} \quad 0^0 = 1?$$

Use limits to justify the answer.

H.14. Evaluate

$$\lim_{x \rightarrow 0} \frac{1 + \frac{1}{2}x^2 - \sqrt{1+x^2}}{(\cos(x) - e^{x^2}) \sin(x^2)}.$$

H.15. Evaluate

$$\lim_{x \rightarrow 0+} \cot(x)^{\sin(x)}.$$

