UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET G

THEME: **Derivatives Theory**

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Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

G.1. Consider the function

$$f(x) = 1/x^2, \quad x \neq 0.$$

Using the limit definition of a derivative show that $f'(-3) = 2/27$.

G.2. Consider the function

$$f(x) = x^3, \quad x \in \mathbb{R}.$$

Using the limit definition of a derivative, obtain $f'(x)$.

G.3. Consider the function

$$f(x) = \sqrt{x}, \quad x > 0.$$

Using the limit definition of a derivative, obtain $f'(x)$.

G.4. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^2, & x \leq 1, \\ x, & x > 1. \end{cases}$$

(a) Draw the graph of f over the interval $[-1, 3]$.

(b) Show that f is differentiable at each $x \neq 1$.

(c) Show that f is not differentiable at $x = 1$.

G.5. Consider the function

$$f(x) = \frac{x}{x+1} \quad \text{for} \quad x \neq -1.$$

Using the limit definition of a derivative, obtain $f'(x)$.

G.6. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^2 + 1, & x < 1, \\ 2x, & x \geq 1. \end{cases}$$

[1] This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

(a) Draw the graph of f over the interval $[-1, 3]$.

(b) Show that f differentiable at $x = 1$ and $f'(1) = 2$.

G.7. Using the ε-δ definitions for the right and left limits of a function on $X \subseteq \mathbb{R}$, prove that the function

$$f(x) = |x|, \quad x \in \mathbb{R},$$

is not differentiable at $x = 0$.

G.8. For $x \in \mathbb{R}$, the sinh (hyperbolic sine) and cosh (hyperbolic cosine) functions are given by

$$\sinh(x) = \tfrac{1}{2}(e^x - e^{-x}) \quad \text{and} \quad \cosh(x) = \tfrac{1}{2}(e^x + e^{-x}).$$

Identities involving these functions are

$$\begin{aligned} &\sinh(a+b) = \sinh(a)\cosh(b) + \cosh(a)\sinh(b) \\ \text{and} \quad &\cosh(a+b) = \cosh(a)\cosh(b) + \sinh(a)\sinh(b) \quad \text{for all } a, b \in \mathbb{R}. \end{aligned}$$

Using the limit definition of a derivative, obtain $\sinh'(x)$ and $\cosh'(x)$. The following limit result may be assumed

$$\lim_{t \to 0} \frac{e^t - 1}{t} = 1.$$

G.9. Prove that the polynomial

$$x^7 + x^5 + x^3 + x + 1000$$

has exactly one root in $\mathbb{R}$.

G.10. Let

$$f(x) = ax^2 + bx + c, \quad x \in \mathbb{R},\ a \neq 0,$$

be a quadratic function of x. Use Rolle's Theorem to prove that f cannot have three roots.

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