UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET F

THEME: **Uniform and Lipschitz Continuity**

Date of this version: **10th July 2026**[1]

Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

F.1. Prove, using the ε-δ definition, that the function $f : [3, 8] \to \mathbb{R}$ given by

$$f(x) = x^2, \quad 3 \leq x \leq 8$$

is uniformly continuous on $[3, 8]$.

F.2. Using the ε-δ definition of a uniformly continuous function, prove that $f : [0, \infty) \to \mathbb{R}$ given by

$$f(x) = \sqrt{x}$$

is uniformly continuous on $[0, \infty)$.

F.3. Using the ε-δ definition of a uniformly continuous function, prove that $f : [0, \infty) \to \mathbb{R}$ given by

$$f(x) = x^2$$

is not uniformly continuous on $[0, \infty)$.

F.4. Using the ε-δ definition of a uniformly continuous function, prove that $f : (0, 5] \to \mathbb{R}$ given by

$$f(x) = 1/x$$

is not uniformly continuous on $(0, 5]$.

F.5. Using only the definition, prove that $f : [2, 5] \to \mathbb{R}$ given by

$$f(x) = 4x^3 - 7x^2$$

is Lipschitz continuous on $[2, 5]$.

[1] This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

F.6. Using the Mean Value Theorem, prove that $f : [2, 5] \to \mathbb{R}$ given by

$$f(x) = 4x^3 - 7x^2$$

is Lipschitz continuous on $[2, 5]$.

F.7. Using only the definition, prove that $f : [-8, 6] \to \mathbb{R}$ given by

$$f(x) = \frac{1}{x + 11}$$

is Lipschitz continuous on $[-8, 6]$.

F.8. Prove that $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \frac{e^x}{(1 + e^x)^2}$$

is Lipschitz continuous on $\mathbb{R}$.

F.9. Prove that $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \frac{x^3 + 2x + 1}{x^4 + 10}$$

is Lipschitz continuous on $\mathbb{R}$.

F.10. Let $f : [0, 1] \to \mathbb{R}$ given by

$$f(x) = \sqrt{x}.$$

(a) Using the ε-δ definition prove that is uniformly continuous on $[0, 1]$.

(b) Using the ε-M definition prove that is not Lipschitz continuous on $[0, 1]$.

♦ ♦ ♦