UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

Exercise Set E

THEME: **Limits and Continuity Theory**

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𝔖𝔬𝔩𝔲𝔱𝔦𝔬𝔫𝔰? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

E.1. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$, prove that

$$\lim_{x\to 6} x^2 = 36.$$

E.2. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$ prove that

$$\lim_{x\to 7} \frac{1}{x} = \frac{1}{7}.$$

E.3. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$, prove that

$$\lim_{x\to 4} x^3 = 64.$$

E.4. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$ prove that

$$\lim_{x\to 7} \frac{1}{x^2+17} = \frac{1}{66}.$$

E.5. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$, and assuming the series result

$$e^x = 1 + x + x/2! + x^3/3! + x^4/4! + \dots \quad \text{for all} \quad x \in \mathbb{R}$$

prove that

$$\lim_{x\to 0} e^x = 1.$$

E.6. Let the function f with domain $\mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0. \end{cases}$$

Using the ε-δ_x definition of a function $f : X \to \mathbb{R}$ being continuous at $x \in X$ prove that f is discontinuous at $x = 0$.

[1] This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

E.7. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$, prove that, for any $c \in \mathbb{R}$,

$$\lim_{x \to c} x^2 = c^2.$$

E.8. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$ prove that, for any $c \neq 0$,

$$\lim_{x \to c} \frac{1}{x} = \frac{1}{c}.$$

E.9. Using the ε-δ definition of a limit for a function on $X \subseteq \mathbb{R}$ prove that, for any $c \in \mathbb{R}$,

$$\lim_{x \to c} x^3 = c^3.$$

E.10. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is an irrational number,} \\ 1 & \text{if } x \text{ is a rational number.} \end{cases}$$

Using the ε-δ_x definition of a function $f : X \to \mathbb{R}$ being continuous, show that f is discontinuous at $x = 3$. The fact that, for any $a, b \in \mathbb{R}$ such that $a < b$, the interval $[a, b)$ contains at least one rational number and one irrational number may be used.

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