UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET D

THEME: Series

Date of this version: **10th July 2026**[1]

Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

D.1. Determine whether or not the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

converges.

D.2. For each $r \in \mathbb{R}$ define the sequence

$$b_k = r^k, \quad k \in \mathbb{N}.$$

Next, define

$$s_k = \sum_{k=0}^{n} b_k, \quad n \in \mathbb{N}$$

to be the sequence of partial sums of b_k.

(a) Show that $s_n = \dfrac{r^{n+1}-1}{r-1}$ for all $r \neq 1$ and $n \in \mathbb{N}$.

(b) Prove that $s_n \begin{cases} \text{converges to } 1/(1-r) \text{ when } |r|<1, \\ \text{diverges when } |r| \geq 1. \end{cases}$

D.3. Determine whether or not the series

$$\sum_{n=1}^{\infty}\frac{1}{4n^2+15}$$

converges.

D.4. Determine whether or not the series

$$\sum_{n=1}^{\infty}\frac{9+\sqrt{n}}{1+2n-n^2}$$

is convergent.

[1]This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

D.5. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{14n^5 + 3n^3}{9n^6 + 7n^2 + 5}$$

converges.

D.6. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(11n + 8)^{1/94}}$$

is convergent. The fact that, for any $\alpha > 0$, the function

$$g(x) = x^{\alpha}$$

is continuous and monotone increasing over $\{x : x > 0\}$ can be assumed.

D.7. Evaluate the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 9}.$$

D.8. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{1}{5n^2 - 837}$$

converges.

D.9. Prove that every absolutely convergent series is convergent.

D.10. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln\big(\ln(15n + 28)\big)}$$

is convergent. The fact that the function $\ln(x)$ is continuous and monotone increasing over $\{x : x > 0\}$ can be assumed.

D.11. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - 62n + 961}$$

converges.

D.12. Let $g \in \mathbb{N}$ be given by

$$g = 10^{10^{100}}.$$

The integer g is relatively famous and known by the name *googolplex*. Determine whether or not the series

$$\sum_{n=0}^{\infty} \frac{g^n}{n!} = \sum_{n=0}^{\infty} \frac{10^{10^{100} n}}{n!}$$

converges.

D.13. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{(33 + 31n^3 + 7n^2)^{1/4}}{(10n^{17} + 11n^{25} + 3n^3 + 98)^{1/14}}$$

converges. The fact that $\sum_{n=1}^{\infty} n^{-\alpha}$ is divergent for $0 < \alpha \leq 1$ and convergent for $\alpha > 1$ may be assumed.

D.14. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

converges.

D.15. Determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - 798n + 113}$$

converges.

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