UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET C

THEME: Sequence Limits

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Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

C.1. Using properties of limits obtain (with $n \in \mathbb{N}$):

(a) $\displaystyle \lim_{n\to\infty} \frac{7n^3 + 2n^2 - n + 16}{4n^3 + 98n^2 + 31n - 11}$,

(b) $\displaystyle \lim_{n\to\infty} \frac{(-1)^n}{\sqrt[3]{19n + 42}}$,

(c) $\displaystyle \lim_{n\to\infty} \frac{1}{\sqrt{n}(\sqrt{n+1} - \sqrt{n})}$.

C.2. Evaluate

$$\lim_{n\to\infty} \frac{974^n}{(n+33)!} \quad (n \in \mathbb{N}).$$

C.3. Obtain

$$\lim_{n\to\infty} \frac{\cos^{703}(n)}{(n+4)^{1/39}} \quad (n \in \mathbb{N}).$$

C.4. (a) Show that $\sqrt{n} > \ln(n)$ for all $n \in \mathbb{N}$. The following result may be assumed: $h : [1, \infty) \to \mathbb{R}$ given by $h(x) = x - 2\ln(x)$ is such that $h(x) > 0$ for all $x > 0$.

(b) Using the Sandwich Theorem, show that

$$\lim_{n\to\infty} \frac{\ln(n)}{n} \to 0.$$

C.5. Evaluate

$$\lim_{n\to\infty} \frac{(3n + 8419)(7n + 2768)(2n + 3452)}{18n^3 + 93275} \quad (n \in \mathbb{N}).$$

[1]This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

C.6. For what values of $x \in \mathbb{R}$ does the sequence

$$f_n(x) = \frac{x + x^n}{1 + x^n}, \quad x \in \mathbb{R},\ n \in \mathbb{N}$$

converge?

C.7. Evaluate the following limit:

$$\lim_{n\to\infty} \left(\sqrt{2n^2 + 7n + 3} - \sqrt{2n^2 - 4n + 9}\right) \quad (n \in \mathbb{N}).$$

C.8. Evaluate the following limit:

$$\lim_{n\to\infty} \left((7n^3 + 93n^2 - 11n + 8)^{1/3} - (7n^3 + 57n^2 - 54n + 29)^{1/3}\right) \quad (n \in \mathbb{N}).$$

C.9. Evaluate

$$\lim_{n\to\infty} \frac{\sqrt[3]{n+2}\,\sqrt[3]{n^2 + 2n + 4}}{(n+1)^2 - (n-1)^2} \quad (n \in \mathbb{N}).$$

C.10. Evaluate

$$\lim_{n\to\infty} \frac{(n+9)!17^{12n+n^2}}{(2n+3)!} \quad (n \in \mathbb{N}).$$

C.11. Evaluate

$$\lim_{n\to\infty} \frac{(n+9)!17^{12n}}{(2n+3)!} \quad (n \in \mathbb{N}).$$

C.12. Using properties of limits obtain

$$\lim_{n\to\infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} \quad (n \in \mathbb{N}).$$

C.13. Evaluate

$$\lim_{n\to\infty} \left\{\left(8 - \frac{1}{\sqrt[n]{2}}\right) \frac{3n^3 + 5}{2n^3 + n^2 - 1}\right\} \quad (n \in \mathbb{N}).$$

C.14. For each $x \in \mathbb{R}$ the hyperbolic tangent function is defined by

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

where

$$\sinh(x) = \tfrac{1}{2}(e^x - e^{-x}) \quad \text{and} \quad \cosh(x) = \tfrac{1}{2}(e^x + e^{-x}), \quad x \in \mathbb{R}.$$

Evaluate

$$\lim_{n\to\infty} \frac{13n^2 + \tanh^{n^2}\left((95n!)^{87014^{n!}}\right) + 7n + 85}{19n^2 + 14n + 10} \quad (n \in \mathbb{N}).$$

C.15. Evaluate

$$\lim_{n\to\infty} \sqrt[n]{3984 + 211^n} \quad (n \in \mathbb{N}).$$

The facts that the functions $\exp$ and $\ln$ are continuous over their domains may be assumed. In addition, the results $\ln(ab) = \ln(a) + \ln(b)$ and $\ln(a^b) = b\ln(a)$ may be assumed.

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