UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences

35007 Real Analysis

EXERCISE SET B

THEME: **Sequences Theory**

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Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

B.1. Let a_n be a sequence such that

$$\lim_{n\to\infty} a_n = a.$$

Using the ε-N definition of the limit of a sequence, prove that for any $c \in \mathbb{R}$,

$$\lim_{n\to\infty} (ca_n) = ca.$$

B.2. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \frac{3n-1}{n+2} = 3.$$

B.3. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \frac{n^3+3}{7n^3+3n} = \frac{1}{7}.$$

B.4. Let a_n be a sequence such that $a_n \geq 0$ for all $n \in \mathbb{N}$ and

$$\lim_{n\to\infty} a_n = a.$$

Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}.$$

B.5. Using the ε-N definition of the limit of a sequence, prove that the sequence

$$a_n = n, \quad n \in \mathbb{N}.$$

does not converge to a limit as $n \to \infty$.

[1]This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

B.6. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty}\left(\sqrt{7n+29}-\sqrt{7n+10}\right)=0.$$

B.7. Using the ε-N definition of the limit of a sequence, prove that the sequence

$$a_n=(-1)^n n, \quad n\in\mathbb{N}.$$

does not converge to a limit as $n\to\infty$.

B.8. Does the sequence

$$a_n=\sin(n)\cos(n^2+1), \quad n\in\mathbb{N}$$

have a convergent subsequence?

B.9. Prove that the sequence

$$\frac{\{1+(-1)^n\}(5n^2+7)}{6n^2+n+3}+\frac{\{1+(-1)^{n+1}\}(19n-11)}{13n^2+8}, \quad n\in\mathbb{N},$$

is bounded and then find a convergent subsequence (which, by the Bolzano-Weierstrass Theorem, must exist).

B.10. Consider the sequence

$$a_n=\frac{19n^2+15n+17}{41n^2-22n-109}, \quad n\in\mathbb{N}.$$

Determine

$$\liminf_{n\to\infty} a_n \quad \text{and} \quad \limsup_{n\to\infty} a_n.$$

B.11. Suppose that the sequence a_n, $n\in\mathbb{N}$, has nth term

$$a_n=2(-1)^{n+1}+\frac{(-1)^n(4n^2+7n-3)}{11n^2-29n}+\frac{23n+5}{11n-7}.$$

Determine

$$\liminf_{n\to\infty} a_n \quad \text{and} \quad \limsup_{n\to\infty} a_n.$$

B.12. Suppose that the sequence a_n, $n\in\mathbb{N}$, has nth term

$$a_n=\sqrt{n}\{1-(-1)^n\}+\frac{\{1+(-1)^n\}(9n^2+14n)}{13n^2+23n^3}.$$

Determine

$$\liminf_{n\to\infty} a_n \quad \text{and} \quad \limsup_{n\to\infty} a_n.$$

B.13. Using the N-ε definition, prove that the sequence

$$a_n = \frac{1}{n}, \quad n \in \mathbb{N},$$

is a Cauchy sequence.

B.14. Using the ε-N definition, prove that

$$a_n = \sqrt[3]{n+5} - \sqrt[3]{n+2}.$$

is a Cauchy sequence.

B.15. Using the N-ε definition, prove that the sequence

$$a_n = n, \quad n \in \mathbb{N}.$$

is not a Cauchy sequence.

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