

UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences
35007 Real Analysis

EXERCISE SET A

THEME: Real Analysis Basic Concepts

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Solutions? *Full solutions to all exercises will be released after the relevant class and/or tutorial.*

A.1. (a) Prove that, for real numbers a, b, c and d ,

$$0 < a < b \quad \text{and} \quad 0 < c < d \quad \text{implies} \quad ac < bd.$$

(b) Using a counterexample show that, for real numbers a, b, c and d ,

$$a < b \quad \text{and} \quad c < d \quad \text{does not imply} \quad ac < bd.$$

A.2. (a) Prove that, for real numbers a and b ,

$$0 < a < b \quad \text{implies} \quad \frac{1}{a} > \frac{1}{b}.$$

(b) Using a counterexample show that, for real numbers a and b ,

$$a < b \quad \text{does not imply} \quad \frac{1}{a} > \frac{1}{b}.$$

A.3. Prove that the set of odd integers is countable.

A.4. For each of the following each pair of sets X and Y and bijection $f : X \rightarrow Y$ obtain the inverse function $f^{-1} : Y \rightarrow X$:

(a) $X = \{\text{bear, goat, swan}\}, Y = \{\text{cub, cygnet, kid}\},$

$$f(\text{bear}) = \text{cub}, \quad f(\text{goat}) = \text{kid}, \quad f(\text{swan}) = \text{cygnet}.$$

(b) $X = \mathbb{R}, Y = (0, 1)$ and $f(x) = e^x / (1 + e^x), \quad x \in X.$

(c) $X = \mathbb{R}, Y = \mathbb{R}$ and $f(x) = \sqrt[5]{9 - 11x}, \quad x \in X.$

¹This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

A.5. Let $A = \{19, -14, 33, 0, 9, -11\}$. Determine

$$\inf A \quad \text{and} \quad \sup A.$$

A.6. Let $\mathbb{P} \subset \mathbb{N}$ denote the set of prime numbers. Determine

$$\inf \mathbb{P} \quad \text{and} \quad \sup \mathbb{P}.$$

A.7. Determine the supremum and the infimum of the set

$$\{m/n : m, n \in \mathbb{N}, m + n \leq 10\}.$$

A.8. Determine the supremum and the infimum of the set

$$\{x - x^2 : 0 < x < 1\}.$$

A.9. Determine the supremum and the infimum of the set

$$\{n/(n+1) : n \in \mathbb{N}\}.$$

A.10. Consider the set $X \subset \mathbb{R}$ given by

$$X = \{x : 8 < x < 14\} = (8, 14).$$

Show that $11\frac{1}{2}$, and 14 are limit points of X and that 6 is an isolated point of X .

A.11. Let $A \subset \mathbb{R}$ be defined by

$$A = \{x : -24 < x < 77\} = (-24, 77).$$

Prove that A is open.

A.12. Let $B \subset \mathbb{R}$ be defined by

$$B = \{x : 4 \leq x \leq 7 \text{ or } x = 9\} = [4, 7] \cup \{9\}.$$

Prove that B is closed.

A.13. Is the empty set $\{\}$ open, closed, both or neither?

A.14. Is $\{79\}$ open, closed, both or neither?

A.15. Prove that the set

$$A = \{x : 3 \leq x < 8\} = [3, 8]$$

is neither open nor closed.

