

UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences
35007 Real Analysis

ASSIGNMENT

Deadline One (see below for explanation): 3:05pm, Wednesday 9th September, 2026.

Deadline Two (see below for explanation): 3:05pm, Wednesday 16th September, 2026.

Preparation instructions: The solutions to the assignment must be handwritten on sheets of paper.

Rules about allowable resources: These are described in detail on subsequent pages of this document.

Submission method: Just before Deadline One and Deadline Two, some boxes will be placed by the lecturer at the front of the classroom. The assignment submission should be placed into one of these boxes.

EXPLANATIONS ABOUT THE DEADLINES:

Deadline One is the official deadline according to the subject's assessment plan. It allows for two weeks of completion time and implies that there is a one week gap between finishing the assignment and participating in Quiz 2. Subject participants are encouraged to submit the assignment by Deadline One and then have more time for Quiz 2 preparation.

Deadline Two corresponds to an automatic one-week extension. This deadline exists due to the fact that some of the subject participants have clashes with the workshop/lectures but must attend the Wednesday 16th September workshop/lecture because of Quiz 2. This allowance is being made for the convenience of some participants. **The price for this convenience is the voiding of any arguments that Quiz 2 preparation is impacted by time spent working on the assignment during the week between Deadline One and Deadline Two.**

ADDITIONAL NOTES:

- Assistance with the assignment is available at the Week 6 and Week 7 tutorials.
- Completion of the assignment is likely to benefit from some of the Solutions for Exercise Sets G, H and I, and these will be released at 1pm on Friday 28th August.

IMPORTANT: ASSIGNMENT RULES

1. Group work is not allowed. Inspection of the solutions of another participant is forbidden.
2. Communication with another subject participant about any aspects of the assignment via speech, e-mail, texting or any other carriage service is not allowed. Discussions about the assignment within the subject tutorials are allowed.
3. Any use at all of generative artificial intelligence platforms such as Google with AI mode and Chat GPT is totally forbidden.
4. The solutions must be written on sheets of paper in legible handwriting.
5. Any notation that is not part of the subject material should not be used. Some examples of non-allowed notation are $\forall$ and $\exists$. The solutions should only use the subject's notation and define any new notation.
6. All solutions must be fully simplified.
7. All arguments must read as sentences with correct grammar and punctuation.

1. Using the ε - δ definition of a limit for a function on $X \subseteq \mathbb{R}$, prove that

$$\lim_{x \rightarrow 10} \frac{22 - x}{x + 7} = \frac{12}{17}.$$

Hint: Exercise Set E and its solutions are relevant to this question.

2. Using only the definition, prove that $f : [-2, 6] \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{2x + 11}{x + 13}$$

is uniformly continuous on $[-2, 6]$.

Hint: Exercise Set F and its solutions are relevant to this question.

3. Determine the values of $x \in \mathbb{R}$ for which the following series converges:

$$\sum_{n=0}^{\infty} n^{143} (x - 18)^n.$$

4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \frac{x}{1 + |x - 2|}, \quad x \in \mathbb{R}.$$

- (a) Show that, for all $h \neq 0$,

$$\frac{f(0 + h) - f(0)}{h} = \frac{1}{1 + |h - 2|}.$$

- (b) Show that, for all $h \neq 0$,

$$\frac{f(2 + h) - f(2)}{h} = \frac{1}{1 + |h|} - \left(\frac{2}{1 + |h|} \right) \frac{|h|}{h}.$$

- (c) Prove that f is differentiable at $x = 0$ and obtain $f'(0)$.

- (d) Prove that f is not differentiable at $x = 2$.

5. Using only the definition, prove that $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = x^{1/5}$$

is not Lipschitz continuous on $\mathbb{R}$.

Hint: Exercise F.10 and its solution are relevant to this question.

6. Obtain the Taylor series for $f(x) = 1/x^2$ about the point $a = 12$ and then determine the values of $x \in \mathbb{R}$ for which the series converges.

The assignment continues overleaf.



7. Evaluate each of the following limits.

(a) $\lim_{x \rightarrow 2187} \frac{x^{1/7} - 3}{x - 2187}.$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(9x^2 + 3x)}{\ln(15x^4 + 11x^3 + 1)}.$

(c) $\lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right).$

For (c), the result $\lim_{x \rightarrow \infty} (e^x - x) = \infty$ can be assumed.

Hint: Exercise Set H and its solutions are relevant to this question.

8. Evaluate

$$\sum_{n=0}^{\infty} \frac{n^4 3^n}{5^n}.$$

Hint: Exercises I.9, I.10 and I.11 and their solutions are relevant to this question.

