UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

Practice Quiz 1 – Solutions

Date of this version: **12th August 2026**[1]

The Floor and Ceiling Notation. These solutions use a less standard notation which is common in mathematical journals and some mathematics textbooks. For $x \in \mathbb{R}$ the floor notation is

$$\lfloor x \rfloor = \text{greatest integer less than or equal to } x = \text{“floor” of } x$$

and the ceiling notation is

$$\lceil x \rceil = \text{smallest integer greater than or equal to } x = \text{“ceiling” of } x.$$

Examples of this notation are

$$\lfloor 5.78 \rfloor = 5, \quad \lceil 5.78 \rceil = 6, \quad \lfloor 34 \rfloor = 34, \quad \lceil 34 \rceil = 34, \quad \lfloor -8.46 \rfloor = -9, \quad \lceil -8.46 \rceil = -8.$$

1. *Relevant page(s) in the notes:* 6

   (a) $\inf A = -43$ and $\sup A = 89$.

   (b) $\inf A = \sqrt{7}$ and $\sup A = 93\sqrt{2}$.

   (c) $\inf A = 0$ and $\sup A = 1$.

   (d) $\inf A = \dfrac{1}{16}$ and $\sup A = 4$.

   (e) $\inf A = -1$ and $\sup A = \dfrac{3}{2}$.

2. *Relevant page(s) in the notes:* 10

   (a) By division of the numerator and denominator by n^3 we have:

$$\lim_{n\to\infty}\left(\frac{2n^3 - 3n}{5n^3 + 4n^2 - 2}\right) = \lim_{n\to\infty}\left(\frac{2 - 3/n^2}{5 + 4/n - 2/n^3}\right) = \frac{2}{5}.$$

   (b) By division of the numerator and denominator by $n^2 = \sqrt{n^4}$ we have:

$$\lim_{n\to\infty}\left(\frac{\sqrt{7n^4 + 12n - 8n^3}}{\sqrt{2n^2 + 9n^4 + 108}}\right) = \lim_{n\to\infty}\left(\frac{\sqrt{7 + 12/n^3 - 8/n}}{\sqrt{2/n^2 + 9 + 108/n^4}}\right) = \sqrt{\frac{7}{9}}.$$

[1] This document is less than a few months old which makes errors more likely. Updated versions with later dates may be released during the semester. Notifications will be sent via e-mail whenever an updated version is released.

(c) By multiplication and division by $\sqrt{n^2+11}+n$ and use of the algebraic identity $(a-b)(a+b) = a^2 - b^2$ we have:

$$\begin{aligned}
\lim_{n\to\infty}\left(\sqrt{n^2+11}-n\right) &= \lim_{n\to\infty}\left\{\frac{\left(\sqrt{n^2+11}-n\right)\left(\sqrt{n^2+11}+n\right)}{\sqrt{n^2+11}+n}\right\} \\
&= \lim_{n\to\infty}\left(\frac{11}{\sqrt{n^2+11}+n}\right) \\
&= \lim_{n\to\infty}\left(\frac{11/n}{\sqrt{1+11/n^2}+1}\right) \\
&= 0.
\end{aligned}$$

3. $\boxed{\textit{Relevant page(s) in the notes: } \text{Supplementary Results}}$

Let

$$a_n \equiv \frac{(2n)!8^n}{(3n)!}.$$

Then all $a_n \neq 0$ and

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(2n+2)!8^{n+1}}{(3n+3)!}\times\frac{(3n)!}{(2n)!8^n} = \frac{8(2n+2)(2n+1)}{(3n+3)(3n+2)(3n+1)} = \frac{(8/n)(2+2/n)(2+1/n)}{(3+3/n)(3+2/n)(3+1/n)}.$$

Then

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(8/n)(2+2/n)(2+1/n)}{(3+3/n)(3+2/n)(3+1/n)} = 0.$$

So by the Ratio Test for Sequences (Theorem S.1 of the Supplementary Results)

$$\lim_{n\to\infty}\frac{(2n)!8^n}{(3n)!} = 0.$$

4. $\boxed{\textit{Relevant page(s) in the notes: } 8}$

For any given $\varepsilon > 0$ set $N = \lceil 4/(49\varepsilon)\rceil$. Then $n \geq N$ implies that

$$n \geq \frac{4}{49\varepsilon} \implies n > \frac{4}{49\varepsilon} - \frac{4}{7} \implies 7n > \frac{4}{7\varepsilon} - 4 \implies 7n+4 > \frac{4}{7\varepsilon} \implies \frac{4}{7(7n+4)} < \varepsilon.$$

Noting that

$$\frac{4}{7(7n+4)} = \left|\frac{n}{7n+4} - \frac{1}{7}\right|$$

we then have

$$n \geq N \implies \left|\frac{n}{7n+4} - \frac{1}{7}\right| < \varepsilon.$$

Therefore, by Definition 1.16,

$$\lim_{n\to\infty}\left(\frac{n}{7n+4}\right) = \frac{1}{7}.$$

Preliminary background working that led to an effective N choice

For each $\varepsilon > 0$ need to get to

$$\left|\frac{n}{7n+4} - \frac{1}{7}\right| < \varepsilon. \tag{1}$$

Simplification of left-hand side of (1) leads to the equivalent condition

$$\frac{4}{7(7n+4)} < \varepsilon \quad \Longleftrightarrow \quad n > \frac{4}{49\varepsilon} - \frac{4}{7}.$$

The simpler condition $n \geq 4/(49\varepsilon)$ implies $n > 4/(49\varepsilon) - (4/7)$ and avoids having to deal with the possibility of negative numbers on the right-hand side (e.g. if $\varepsilon = 1$). So setting

$$N = \lceil 4/(49\varepsilon) \rceil = \text{the smallest integer greater than or equal to } 4/(49\varepsilon)$$

will allow us to say that $n \geq N$ implies (1).

♦ ♦ ♦