35007

Real Analysis

Professor Matt Wand

University of Technology Sydney

Main classroom rule next...

First Up

Five paper handouts

(out of six for the whole) subject)

You Have Just Entered a ...

SCREEN-FREE ZONE

Classroom Rules

- NO switched on mobile phones.
- NO texting.
- NO *exting.
- NO open iPads.
- NO iPods.
- NO other iDevices.
- NO open laptops.

Classroom Rules (continued)

- NO tindering.
- NO you-tubing.
- NO tumblring.
- NO grindring.
- NO fortniting.
- NO tik-toking.
- NO hinge-ing.
- NO reddit-ing.

Classroom Rules (continued)

- NO facebooking.
- NO tweeting.
- NO X-ing.
- NO googling.
- NO snapchatting.
- NO instagramming.
- NO google glassing.

Classroom Rules (continued)

- NO chat GPT-ing.
- NO grok-ing.
- NO gemini-ing.
- NO copilot-ing.
- NO claude-ing.
- NO meta glass-ing.

Yes, this class is a ...

SCREEN-FREE ZONE

Clarifications?

Just to be sure... that's NO to:

`switched on mobile phones, texting, *exting,`
`open iPads, iPods, other iDevices, open laptops,`
`facebooking, tweeting, X-ing, googling,`
`snapchatting, instagramming, google glassing,`
`tindering, you-tubing, tumblring, grindring,`
`fortniting, tik-toking, hinge-ing, reddit-ing,`
`chat GPT-ing, grok-ing, gemini-ing, copilot-ing,`
`claude-ing, meta-glassing.`

Learning Strategy

- ***The 150 exercises and solutions (learning by doing!).***
- ***The notes and supplement.***
- ***Class exercises and discussion.***
- ***Some, but not many, slides.***
- ***Tutorials with Brian and Sam.***

The Classes and Tutorials
Partial Misalignment Problem

Week 1	Week 2	Week 3	Week 4
	Tutes for Class 1 Wed. morning	Tutes for Class 2 Wed. morning	Tutes for Class 3 Wed. morning
Class 1 Wed. aft'noon	Class 2 Wed. aft'noon	Class 3 & Quiz 1 Wed. aft'noon	Class 4 Wed. aft'noon
	Tute for Class 1 Fri. morning	Tute for Class 2 Fri. morning	Tute for Class 3 Fri. morning

Lecturer's History

Born in Wollongong too long ago.

Undergradate major in Pure Maths and
Statistics at Univ. Wollongong (early 1980s).

First learnt Real Analysis in 1983 – only 43 years ago.

PhD in Statistics at Aust. National Univ. (late 1980s).

Lecturing jobs in Texas, U.S.A. (early 1990s).

Lecturing job at Univ. New South Wales (mid 1990s).

Associate Professor at Harvard Univ., U.S.A. (1997-2002).

Professor of Statistics at Univ. New South Wales (2003-2006).

Research Professor of Statistics at Univ. Wollongong (2007-2010).

Disintinguished Professor of Statistics at UTS (2011-today).

inf A

and

sup A

Lecturer's 2026 Published Result

(A1) The cell dimensions m and m' diverge to ∞ in such a way that $m = O(m')$ and $m' = O(m)$.

(A2) The within-cell sample sizes $n_{ii'}$ diverge to ∞ in such a way that

$$\max_{1\le i\le m,\ 1\le i'\le m'} |n_{ii'}/n - C_{ii'}| \to 0 \quad \text{as } m, m' \to \infty$$

for positive constants $C_{ii'}$, $1 \le i \le m$, $1 \le i' \le m'$, that are bounded above and away from zero. Also, $n/m \to 0$ as m and n diverge.

(A3) All entries of both $\boldsymbol{X}_{\mathrm{A}\circ}$ and $\boldsymbol{X}_{\mathrm{B}\circ}$ are not degenerate at zero and have finite second moment.

Result 3.1: Assume that (A1)–(A3) and some additional regularity conditions hold. Then

$$\begin{bmatrix} \left\{\dfrac{\boldsymbol{\Sigma}^0}{m} + \dfrac{(\boldsymbol{\Sigma}')^0}{m'}\right\}^{-1/2}\left(\hat{\boldsymbol{\beta}}_{\mathrm{A}} - \boldsymbol{\beta}_{\mathrm{A}}^0\right) \\ \left\{\dfrac{(\sigma^2)^0\boldsymbol{C}_{\boldsymbol{\beta}_{\mathrm{B}}}}{mm'n}\right\}^{-1/2}\left(\hat{\boldsymbol{\beta}}_{\mathrm{B}} - \boldsymbol{\beta}_{\mathrm{B}}^0\right) \\ \left\{\dfrac{2\boldsymbol{D}_{d_{\mathrm{A}}}^{+}(\boldsymbol{\Sigma}^0\otimes\boldsymbol{\Sigma}^0)\boldsymbol{D}_{d_{\mathrm{A}}}^{+\top}}{m}\right\}^{-1/2}\mathrm{vech}(\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}^0) \\ \left\{\dfrac{2\boldsymbol{D}_{d_{\mathrm{A}}}^{+}((\boldsymbol{\Sigma}')^0\otimes(\boldsymbol{\Sigma}')^0)\boldsymbol{D}_{d_{\mathrm{A}}}^{+\top}}{m'}\right\}^{-1/2}\mathrm{vech}(\hat{\boldsymbol{\Sigma}}' - (\boldsymbol{\Sigma}')^0) \\ \left[\dfrac{2\{(\sigma^2)^0\}^2}{mm'n}\right]^{-1/2}\left(\hat{\sigma}^2 - (\sigma^2)^0\right) \end{bmatrix} \xrightarrow{D} N(\boldsymbol{0}, \boldsymbol{I}).$$

See matt-p-wand.net for full details, including a 28 page proof!

Infimum / Supremum Example 1

infimum (inf) is like minimum (min) and

supremum (sup) is like maximum (max)

$$\inf\{1,2,3,4,5\} = \min\{1,2,3,4,5\} = 1$$

$$\sup\{1,2,3,4,5\} = \max\{1,2,3,4,5\} = 5$$

Infimum / Supremum Example 2

$$A = \{x : 4 \le x \le 9\} = [4, 9]$$

But inf is often not the same as min

and sup is often not the same as max.

Infimum / Supremum Example 2

$$A = \{x : 4 \le x \le 9\} = [4, 9]$$

$\inf A = 4$ and $\sup A = 9$

But inf is often not the same as min

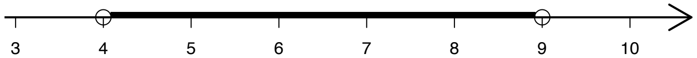
and sup is often not the same as max.

$\inf A =$ greatest lower bound on elements of A

$\sup A =$ least upper bound on elements of A

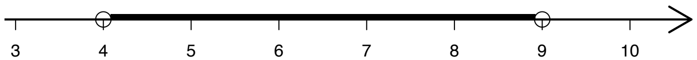
Infimum / Supremum Example 3

$$B = \{x : 4 < x < 9\} = (4, 9)$$



Infimum / Supremum Example 3

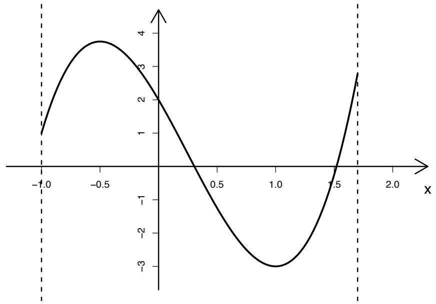
$$B = \{x : 4 < x < 9\} = (4, 9)$$



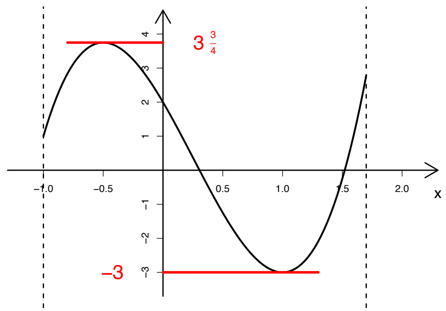
$\inf B = 4$ and $\sup B = 9$

Infimum / Supremum Example 4

$$C = \{4x^3 - 3x^2 - 6x + 2 : -1 \le x < 1.7\}$$



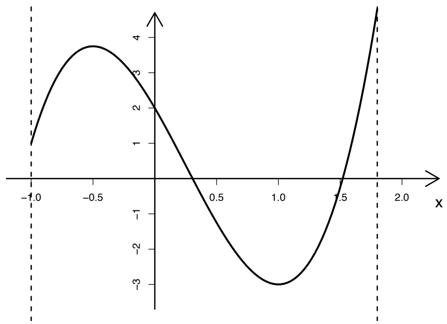
Infimum / Supremum Example 4 (continued)



$\inf C = -3$ and $\sup C = 3\frac{3}{4}$

Infimum / Supremum Example 5

$$D = \{4x^3 - 3x^2 - 6x + 2 : -1 \leq x < 1.8\}$$



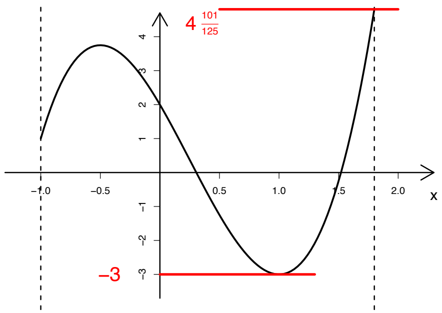
ASIDE: The Definition of $\mathbb{N}$

In this subject and throughout Australia and e.g. the United States of America:

$\mathbb{N}$ = the set of natural numbers $= \{1, 2, 3, \ldots\}$.

But in some parts of the world 0 (zero) is included in the set of natural numbers.

Infimum / Supremum Example 5 (continued)



$\inf D = -3$ and $\sup D = 4\frac{101}{125}$

ASIDE: The Definition of $\mathbb{N}$

In this subject and throughout Australia and e.g. the United States of America:

$\mathbb{N}$ = the set of natural numbers $= \{1, 2, 3, \ldots\}$.

But in some parts of the world 0 (zero) is included in the set of natural numbers.

Please note:* *We are not including 0 in $\mathbb{N}$ throughout 35007 Real Analysis.

Infimum / Supremum Class Exercise

Write down $\inf A$ and $\sup A$ in each case.

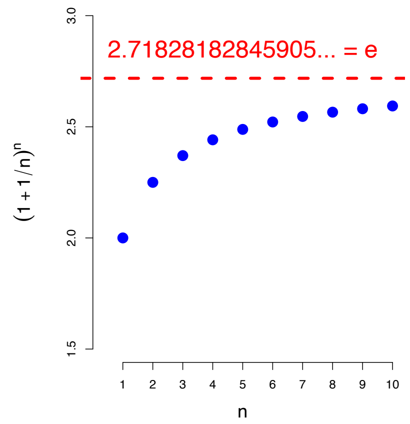
1 $A = \{1, 1/2, 1/3, 1/4\}$

2 $A = \{1, 1/2, 1/3, 1/4, 1/5, \ldots\} = \{1/n : n \in \mathbb{N}\}$

★ 3 $A = \{(1 + 1/n)^n : n \in \mathbb{N}\}$

(★ = challenging and possibly unfair.)

Class Exercise SOLUTION of 3



$\inf\{(1 + 1/n)^n : n \in \mathbb{N}\} = 2$ and $\sup\{(1 + 1/n)^n : n \in \mathbb{N}\} = e.$

Class Exercise SOLUTIONS

1 $\inf\{1, 1/2, 1/3, 1/4\} = 1/4,$

$\sup\{1, 1/2, 1/3, 1/4\} = 1.$

2 $\sup\{1, 1/2, 1/3, 1/4, 1/5, \ldots\} = 1,$

$\inf\{1, 1/2, 1/3, 1/4, 1/5, \ldots\} = 0.$

The Most Relevant Exercises (from the subject exercise sets)

A.5, A.6, A.7, A.8 and A.9.

♦ ♦ ♦