YOU'RE INVITED

TO

QUIZ 1 OF 35007 REAL ANALYSIS

WHEN? 3:10pm to 4:10pm, Wednesday 12th August 2026.

WHERE? Room 320, Level 3, Building 4.

Details ...

Quiz 1 Timing Explanation

There is a requirement by all lecturers to provide assessment-based feedback on or before Week 4 of the semester. Having Quiz 1 in Week 3 allows this requirement to be met.

Examinable Material

The examinable material for Quiz 1 is:

- The infimum and supremum concepts.
- Sequences theory and properties.

The relevant parts of main notes and supplementary notes are:

Lemma 1.10	Definition 1.11	Definition 1.12
Definition 1.15	Definition 1.16	Theorem 1.21
Theorem 1.23	Theorem S.1.	

Permitted and Forbidden Items

The permitted and forbidden items for Quiz 1 is conveyed by the following rules that will appear on the first page of the quiz document:

Items allowed:
Pens One A4 sheet of paper with summaries of relevant material from the notes. Both sides of the sheet can contain material.

Items not allowed:
Please leave all items with electronics such as mobile phones, electronic watches, ear phones and AI glasses, switched off and store in accompanying baggage.

PRACTICE EXERCISES

Out of Exercise Sets A–L, the following have possible relevance to Quiz 1.

A.5.	A.6.	A.7.	A.8.	A.9.
B.1.	B.2.	B.3.	B.4.	B.5.
C.1.	C.2.	C.3.	C.5.	C.7.
C.11.	C.12.			

It follows that these exercises are recommended as good practice ones for Quiz 1. For convenience, the recommended practice exercises are re-listed below.

A.5. Let $A = \{19, -14, 33, 0, 9, -11\}$. Determine

$$\inf A \quad \text{and} \quad \sup A.$$

A.6. Let $\mathbb{P} \subset \mathbb{N}$ denote the set of prime numbers. Determine

$$\inf \mathbb{P} \quad \text{and} \quad \sup \mathbb{P}.$$

A.7. Determine the supremum and the infimum of the set

$$\{m/n : m, n \in \mathbb{N}, m + n \leq 10\}.$$

A.8. Determine the supremum and the infimum of the set

$$\{x - x^2 : 0 < x < 1\}.$$

A.9. Determine the supremum and the infimum of the set

$$\{n/(n+1) : n \in \mathbb{N}\}.$$

B.1. Let a_n be a sequence such that

$$\lim_{n\to\infty} a_n = a.$$

Using the ε-N definition of the limit of a sequence, prove that for any $c \in \mathbb{R}$,

$$\lim_{n\to\infty} (ca_n) = ca.$$

B.2. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \frac{3n-1}{n+2} = 3.$$

B.3. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \frac{n^3+3}{7n^3+3n} = \frac{1}{7}.$$

B.4. Let a_n be a sequence such that $a_n \geq 0$ for all $n \in \mathbb{N}$ and

$$\lim_{n\to\infty} a_n = a.$$

Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}.$$

B.5. Using the ε-N definition of the limit of a sequence, prove that the sequence

$$a_n = n, \quad n \in \mathbb{N}.$$

does not converge to a limit as $n \to \infty$.

C.1. Using properties of limits obtain (with $n \in \mathbb{N}$):

1. $\lim_{n\to\infty} \dfrac{7n^3+2n^2-n+16}{4n^3+98n^2+31n-11}$,

2. $\lim_{n\to\infty} \dfrac{(-1)^n}{\sqrt[3]{19n+42}}$,

3. $\lim_{n\to\infty} \dfrac{1}{\sqrt{n}(\sqrt{n+1}-\sqrt{n})}$.

C.2. Evaluate

$$\lim_{n\to\infty} \frac{974^n}{(n+33)!} \quad (n \in \mathbb{N}).$$

C.3. Obtain

$$\lim_{n\to\infty} \frac{\cos^{703}(n)}{(n+4)^{1/39}} \quad (n \in \mathbb{N}).$$

C.5. Evaluate

$$\lim_{n\to\infty} \frac{(3n+8419)(7n+2768)(2n+3452)}{18n^3+93275} \quad (n \in \mathbb{N}).$$

C.7. Evaluate the following limit:

$$\lim_{n\to\infty} \left(\sqrt{2n^2+7n+3} - \sqrt{2n^2-4n+9}\right) \quad (n \in \mathbb{N}).$$

C.11. Evaluate

$$\lim_{n\to\infty} \frac{(n+9)!17^{12n}}{(2n+3)!} \quad (n \in \mathbb{N}).$$

C.12. Using properties of limits obtain

$$\lim_{n\to\infty} \frac{(-2)^n+3^n}{(-2)^{n+1}+3^{n+1}} \quad (n \in \mathbb{N}).$$

A PDF file containing solutions to the practice exercises will be made available on `matt-p-wand.net/35007.html` at 4:00pm Tuesday 4th August.

UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

35007 Real Analysis

Practice Quiz 1

Time allowed: 1 hour

Items allowed:
Pens One A4 sheet of paper with summaries of relevant material from the notes. Both sides of the sheet can contain material.

Items not allowed:
Please leave all items with electronics such as mobile phones, electronic watches, ear phones and AI glasses, switched off and store in accompanying baggage.

Instructions [1]

1. The quiz questions are overleaf. Please wait until instructed to overturn this sheet and start reading them.
2. Write your solutions in the examination booklet. Please make sure that your name and student number is written on the first page of the booklet.
3. Unless stated otherwise, all working must be shown and partial credit will be awarded for correct steps in your working.
4. Additional background working that is not part of your solutions should be clearly marked as being separate from your solutions.
5. You may leave before the end of the permitted time. Please do so quietly and place your examination booklet in the box at the front of the room.

[1] These instructions and the above restrictions don't apply to the practice quiz (unless you want them to) and are given here to show you how the actual quiz will be.

1. For each of the following definitions of the set A write down

$$\inf A \quad \text{and} \quad \sup A.$$

For this question working does not need to be shown.

(a) $A = \{85, -41, 32, 89, -43, 56\}$.

(b) $A = \{x : \sqrt{7} < x < 93\sqrt{2}\}$.

(c) $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$.

(d) $A = \{m/n^2 : m, n \in \mathbb{N},\ m + n = 5\}$.

(e) $A = \left\{\frac{1}{n} + (-1)^n : n \in \mathbb{N}\right\}$.

2. In this question you may assume properties of limits such as

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n} \quad \text{and} \quad \lim_{n\to\infty} \sqrt{a_n} = \sqrt{\lim_{n\to\infty} a_n}.$$

Evaluate and fully simplify each of:

(a) $\lim\limits_{n\to\infty} \left(\dfrac{2n^3 - 3n}{5n^3 + 4n^2 - 2}\right)$.

(b) $\lim\limits_{n\to\infty} \left(\dfrac{\sqrt{7n^4 + 12n - 8n^3}}{\sqrt{2n^2 + 9n^4 + 108}}\right)$.

(c) $\lim\limits_{n\to\infty} \left(\sqrt{n^2 + 11} - n\right)$.

3. Using one or more theorems from the subject material evaluate

$$\lim_{n\to\infty} \frac{(2n)!8^n}{(3n)!}.$$

4. Using the ε-N definition of the limit of a sequence, prove that

$$\lim_{n\to\infty} \left(\frac{n}{7n + 4}\right) = \frac{1}{7}.$$

END OF QUIZ

A PDF file containing solutions to Practice Quiz 1 will be made available on `matt-p-wand.net/35007.html` at 4:00pm Tuesday 4th August.