

Notification and information regarding

THE FINAL EXAMINATION OF 35007 REAL ANALYSIS

TIME/DATE: 3:10pm to 4:40pm, Wednesday 21st October 2026.

LOCATION: Room 320, Level 3, Building 4.

Important notice about ID cards:
Checking of UTS student identification cards will take place for the final examination. All participants are advised to <i>bring their UTS ID card</i> to the final examination.

EXAMINABLE MATERIAL

The examinable material for the final examination corresponds to the entire 2026 Spring Semester running of 35007 Real Analysis. This is reflected in the slide presentations, exercise sets and solutions, supplementary results document and the subject notes – but with some exclusions, which are described in the next paragraph.

Segments of the subject notes that are excluded from the examinable material are those which do not feature in any of the slide presentations or the exercise sets. Specific exclusions are:

Theorem 1.9	Theorem 1.19	Theorem 1.22
Theorem 1.25	Proposition 1.29	Theorem 1.47
Definition 2.14	Theorem 2.15	Theorem 2.18
Corollary 2.20	Theorem 2.22	Corollary 3.12
Proposition 3.13	Theorem 3.14	Theorem 3.18
Theorem 3.19	Definition 3.20	Proposition 3.21
Theorem 3.22	Theorem 3.23	Theorem 3.24
Proposition 3.25	Theorem 3.26	Theorem 3.35
Theorem 3.36	Proposition 4.1	Theorem 4.4
Theorem 4.7	Theorem 5.8	Definition 5.11
Theorem 5.12	Theorem 5.13	Theorem 5.14
Theorem 5.15	Lemma 5.16	

Theorem 3.34 (Taylor's Theorem) features in the exercise sets and slide presentations. However, due to its detailed nature, Theorem 3.34 is also excluded from the final examination material.

Note that Theorem S.1 and Theorem S.2 in the supplementary results are included in the examinable material.

Proofs of results in the subject material are not examinable.

Exclusion of a result or definition does not imply lesser importance. Its exclusion reflects the limitations of the amount of material that can be reasonably covered to a level of examinability during a 12 weeks subject.

PERMITTED AND FORBIDDEN ITEMS

The permitted and forbidden items for the final examination is conveyed by the following rules that will appear on the first page of the examination document:

Items allowed:
UTS student identification card (required) Pens Two A4 sheets of paper with summaries of relevant material from the notes. Both sides of the sheets can contain material.

Items not allowed:
Please leave all items with electronics such as calculators, mobile phones, electronic watches, ear phones and AI glasses, switched off and stored in accompanying baggage.

UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences
35007 Real Analysis

Practice Final Examination

Time allowed: 1 hour and 30 minutes

Items allowed:

UTS student identification card (required)
Pens
Two A4 sheets of paper with summaries
of relevant material from the notes.
Both sides of the sheets can contain material.

Items not allowed:

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such as calculators, mobile phones,
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Instructions ¹

1. The final examination questions are overleaf. Please wait until instructed to overturn this sheet and start reading them.
2. Write your solutions in the examination booklet. Please make sure that your name and student number is written on the first page of the booklet.
3. Unless stated otherwise, all working must be shown and partial credit will be awarded for correct steps in your working.
4. Additional background working that is not part of your solutions should be clearly marked as being separate from your solutions.
5. You may leave before the end of the permitted time. Please do so quietly and place your examination booklet in the box at the front of the room.

¹These instructions and the above restrictions don't apply to the practice final examination (unless you want them to) and are given here to show you how the actual final examination will be.

1. For this question only, working does not need to be shown.

(a) For each of the following definitions of the set A write down

$$\inf A \quad \text{and} \quad \sup A.$$

i. $A = \left\{ \frac{2}{3-7n} : n \in \mathbb{N} \right\}.$

ii. $A = \{(x-3)^2 : -1 < x < 4\}.$

(b) For each of the following definitions of the sequence $\{a_n : n \in \mathbb{N}\}$ write down

$$\liminf_{n \rightarrow \infty} a_n \quad \text{and} \quad \limsup_{n \rightarrow \infty} a_n.$$

i. $a_n = \frac{11(-1)^{n+1}n^2}{4n^2 + 9n + 6}.$

ii. $a_n = \frac{e^n + 8}{5e^n + 14}.$

iii. $a_n = \frac{n^5 I(n \in \mathbb{P}) - 3n^5 I(n \notin \mathbb{P})}{7n^5 + 4n^2 - 27}$ where $I(\mathcal{P}) = 1$ if $\mathcal{P}$ is true, $I(\mathcal{P}) = 0$ if $\mathcal{P}$ is false and $\mathbb{P}$ is the set of prime numbers.

2. For each $n \in \mathbb{N}$, let $f_n : [0, \infty) \rightarrow \mathbb{R}$ be given by

$$f_n(x) = \sum_{k=1}^n \frac{(-1)^{2k+1} e^{x/k}}{(1 + 2e^{x/k})k\sqrt{k}}.$$

Prove that f_n is uniformly convergent on $[0, \infty)$. The fact that $\sum_{k=1}^{\infty} \frac{1}{k^\alpha}$ diverges for $\alpha \leq 1$ and converges for $\alpha > 1$ may be assumed.

3. This question uses the m_i , M_i , $L(f, \mathcal{P})$ and $U(f, \mathcal{P})$ notations from the subject material. Let $f : [0, 1] \rightarrow \mathbb{R}$ be given by

$$f(x) = \frac{1}{x^2 + 9}, \quad 0 \leq x \leq 1.$$

For the following family of partitions of $[0, 1]$:

$$\mathcal{P} = \left\{ 0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1 \right\}, \quad n \in \mathbb{N},$$

it may be shown that the m_i and M_i quantities are

$$m_i = \frac{1}{i^2/n^2 + 9} \quad \text{and} \quad M_i = \frac{1}{(i-1)^2/n^2 + 9}, \quad i = 1, \dots, n.$$

(a) Show that

$$U(f, \mathcal{P}) - L(f, \mathcal{P}) = \frac{1}{90n}.$$

(b) Prove that f satisfies Riemann's Criterion on $[0, 1]$.

The final examination continues overleaf.



4. Theorem 2.19 in the subject material may be relevant to this question. It states:

*Suppose $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ and $f(a)f(b) < 0$.
Then there is a $c \in [a, b]$ such that $f(c) = 0$.*

Using Rolle's Theorem, prove that $f : [3, 8] \rightarrow \mathbb{R}$ given by

$$f(x) = 4x - 27, \quad 3 \leq x \leq 8,$$

has exactly one root in the interval $[3, 8]$.

5. Evaluate each of the following limits.

(a) $\lim_{x \rightarrow 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}.$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(2x^6 + 8x^5)}{\ln(5x^3 + 943)}.$

(c) $\lim_{x \rightarrow \infty} x^{1/x}.$

6. For each $n \in \mathbb{N}$ let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f_n(x) = \frac{x^2}{3 + 4x^2 + \{2 + (-1)^n\}/n}, \quad x \in \mathbb{R}.$$

Also let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \frac{x^2}{3 + 4x^2} \quad x \in \mathbb{R}.$$

Prove that

f_n converges uniformly to f on $\mathbb{R}$.

END OF FINAL EXAMINATION

A PDF file containing solutions to Practice Final Examination will be made available by

matt-p-wand.net/35007.html at 4:00pm Friday 9th October 2026.