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CLASS 9; LIVE INTEGRATION
BY SUBSTITUTION; EXAMPLE 1

$$I_1 = \int_0^5 x^3 \sqrt{x^4 + 1} \, dx.$$

Let

$$u = x^4 + 1 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow du = 4x^3 dx.$$

Also,

$$x = 0 \Rightarrow u = 1 \text{ and } x = 5 \Rightarrow u = 626.$$

Then

$$I_1 = \frac{1}{4} \int_0^5 \sqrt{x^4 + 1} \cdot 4x^3 dx$$

$$= \frac{1}{4} \int_1^{626} u^{1/2} du = \frac{1}{4} \left[\frac{u^{3/2}}{3/2} \right]_1^{626}$$

$$= \frac{1}{6} (626^{3/2} - 1).$$

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BY SUBSTITUTION ; EXAMPLE 2

$$I_2 = \int_0^{\infty} x^7 e^{-13x^4} dx$$

Let

$$u = 13x^4 \Rightarrow x = \frac{u^{1/4}}{13^{1/4}} \Rightarrow \frac{dx}{du} = \frac{\frac{1}{4} u^{-3/4}}{13^{1/4}}$$

$$\Rightarrow dx = \frac{1}{4(13^{1/4})} u^{-3/4} du$$

Also

$$x=0 \Rightarrow u=0 \quad \text{and} \quad x=\infty \Rightarrow u=\infty$$

Then

$$\begin{aligned} I_2 &= \int_0^{\infty} \left(\frac{u^{1/4}}{13^{1/4}} \right)^7 \cdot e^{-u} \cdot \frac{1}{4(13^{1/4})} u^{-3/4} du \\ &= \frac{1}{4(13^2)} \int_0^{\infty} u e^{-u} du = \frac{1}{4(169)} \int_0^{\infty} u^{2-1} e^{-u} du \\ &= \frac{\Gamma(2)}{676} = \frac{1!}{676} = \frac{1}{676} \end{aligned}$$